

LANGLEY
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A TWO DIMENSIONAL POWER SPECTRAL ESTIMATE
FOR SOME NONSTATIONARY PROCESSES

73 p.

A Thesis
Presented to
The Graduate College
Hampton University

In Partial Fulfillment
of the Requirements for the Degree

Master of Science

(NASA-CR-186100) A TWO DIMENSIONAL POWER
SPECTRAL ESTIMATE FOR SOME NONSTATIONARY
PROCESSES M.S. Thesis (Hampton Univ.) 73 p
CSCL 12A

N90-14843

Unclas

G3/64 0252529

by

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December 1989

NAG1-768

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ACKNOWLEDGEMENTS

I wish to express my thanks to Lynwood T. Smith, my father, Almeta L. Smith, my mother, Amonia, my sister, Lynwood and Vernon, my brothers, for their love and support during my graduate studies.

The faculty of the Department of Mathematics for their interest and encouragement in my course work and research.

The NASA Langley Research Center whose grant I was supported under (NAG-1-768).

Most of all, I wish to express my deep gratitude to Dr. Abol Ghassem Miamee, my advisor, and Dr. Jay C. Hardin, my NASA contact, without whose patience, help and genuine concern no part of this thesis would have been possible.

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CHAPTER I

INTRODUCTION

In recent years interest has grown in the use of nonstationary random processes for modelling physical phenomena. This is due to the fact that many physical phenomena display nonstationary behavior. Currently these phenomena are generally modelled as stationary because all aspects of the theory of stationary processes e.g. prediction and spectral representation are quite well developed [1] and [7]. However, if these phenomena are to be better understood, they must be modeled as nonstationary. One aspect which distinguishes a nonstationary process from a stationary process is the power spectral density. Nonstationary processes have a two dimensional spectral representation while stationary processes have a one dimensional spectral representation. For nonstationary processes the theory is not complete, some basic questions such as interpretations of the spectral representation still need to be investigated.

In this thesis, a two dimensional estimate for the power spectral density of a nonstationary process will be developed. The estimate will be applied to helicopter noise data which is clearly nonstationary. The acoustic pressure from the

isolated main rotor and isolated tail rotor is known to be periodically correlated (PC) and the combined noise from the main and tail rotors is assumed to be correlation autoregressive (CAR). The results of this nonstationary analysis will be compared with the current method of assuming that the data is stationary and analyzing it as such. Another method of analysis is to introduce a random phase shift into the data as shown by Papoulis [8] to produce a time history which can then be accurately modeled as stationary. This method will also be investigated for the helicopter data in this thesis.

The chapters of this thesis will be outlined as follows: the remainder of chapter I discusses background material necessary for understanding the development of this thesis. Chapter II discusses a method used to determine the period of a PC process when the period is not known. The period of a PC process must be known in order to produce an accurate spectral representation for the process. In chapter III, the spectral estimate is actually developed. The bias and variability of the estimate are also discussed. Finally, in chapter IV, the current method for analyzing nonstationary data is compared to that of using a two dimensional spectral representation. In addition, the method of phase shifting the data is examined. Conclusions are then made regarding the comparison of these methods.

A. Preliminaries and Background

The purpose of this section is to review basic definitions and to introduce the classes of stochastic processes which will be investigated in this thesis.

Definitions

The intent here is to give a review of some fundamental concepts and definitions of probability theory and stochastic process theory.

The probability space associated with a random experiment consists of three items (S, β, P) :

1. S is the sample space containing all possible outcomes ζ .
2. β is the collection of all events or subsets of S .
3. P is the probability measure defined on β .

A random variable on a probability space (S, β, P) is any β -measurable function X that usually maps the sample space S into the real line R (although the function X can take complex values). Any real random variable X has a distribution function F_x defined by

$$F_x(\alpha) = P\{\zeta \in S: X(\zeta) \leq \alpha\}, \quad \text{for all } \alpha \in R.$$

The distribution function has four basic properties.

1. $F_x(+\infty) = 1$
2. $F_x(-\infty) = 0$
3. $F_x(\alpha_1) \leq F_x(\alpha_2)$ for $\alpha_1 \leq \alpha_2$
4. F_x is continuous from the right, that is $\lim_{\alpha \rightarrow \alpha_0} F_x(\alpha) = F_x(\alpha_0^+) = F_x(\alpha_0)$.

If it exists, there is also a density function p_x associated with a random variable X such that

$$F_x(\alpha) = \int_{-\infty}^{\alpha} p_x(\gamma) d\gamma, \quad \text{for all } \alpha \in \mathbb{R}.$$

The density function $p_x(\alpha) \geq 0$ for all $\alpha \in \mathbb{R}$.

The moments of a random variable X are defined by

$$m_x^k = E(X^k) = \int_{-\infty}^{\infty} \alpha^k p_x(\alpha) d\alpha, \quad k=0,1,2,\dots,$$

where k is the order of the moment. The central moments of X are defined as follows

$$\eta_x^k = E\{(X-\mu)^k\} = \int_{-\infty}^{\infty} (\alpha-\mu)^k p_x(\alpha) d\alpha, \quad k=0,1,2,\dots,$$

where k is the order of the central moment. Here, the first order moment $m_x^1 = \mu$ is called the mean or expected value. And the second order central moment is called the variance of X .

A stochastic process or random process is a family of random variables $\{X(\lambda, \zeta), \lambda \in \Lambda\}$, where Λ is the index set of the parameter λ . Usually the index set Λ is either the set $Z = \{0, \pm 1, \pm 2, \dots\}$ of integers in which case the process is called discrete, or the set \mathbb{R} of real numbers, in which case the process is called continuous. Although the developments made in this work are for a continuous process, the actual data used to produce the spectral estimate is discrete, due to the necessity of sampling.

The moments of a stochastic process are defined similar to those of a random variable except that they now depend on

time. The first moment is now called the mean function, and the second central moment is called the variance function of the process.

B. Classes of Stochastic Processes

Let $X(t)$ be a stochastic process with a finite second moment whose mean is zero and let

$$R_x(t_1, t_2) = E\{X(t_1)X(t_2)\}.$$

This relationship is called the autocorrelation of the process.

A stochastic process $X(t)$ is called stationary if $R_x(t_1, t_2)$ depends only on $t_2 - t_1$. Therefore,

$$R_x(t_1, t_2) = R_x(t_1 + t, t_2 + t) \quad \text{for all } t_1, t_2 \in \mathbb{R}$$

where t is an arbitrary time. Stationary processes are the most well-known process and have been thoroughly developed, see for example [1]. In general any process which is not stationary is called nonstationary. Stationary processes are a subclass of the following nonstationary processes: PC, CAR, and harmonizable processes. All stationary processes have a spectral representation, a fact, which is very useful for obtaining information about the process.

A process $X(t)$ is called periodically correlated (PC) if there exists a time $T \geq 0$ such that

$$R_x(t_1, t_2) = R_x(t_1 + T, t_2 + T) \quad \text{for all } t_1, t_2 \in \mathbb{R}.$$

The smallest such T is called period of the process. A class of processes called correlation autoregressive (CAR) was introduced by Miamee and Hardin in [5] and [3]. CAR processes

are defined to be those processes for which there exist finitely many scalars a_j such that

$$R_x(t_1, t_2) = \sum_j a_j R(t_1 + Y_j, t_2 + Y_j), \text{ for all } t_1, t_2 \in \mathbb{R}$$

where the Y_j 's are fixed times.

A random process $X(t)$ is called harmonizable if the double Fourier transform of its autocorrelation exists. That is

$$(1) \quad S_x(\omega_1, \omega_2) = 1/4\pi^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 R_x(t_1, t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)}.$$

If this is the case, then $S_x(\omega_1, \omega_2)$ is called the power spectral density of the process $X(t)$. All of the above classes of processes have some members which are harmonizable (see Figure 1). For a stationary process, replacing $R(t_1, t_2)$ by $R(t_2 - t_1)$ and substituting $t_2 - t_1$ by τ allows (1) to simplify to

$$S_x(\omega_1, \omega_2) = S_x\{(\omega_1 + \omega_2)/2\} \delta(\omega_2 - \omega_1)$$

where $\delta(\cdot)$ is the Dirac delta function, and

$$S_x(\omega) = 1/2\pi \int_{-\infty}^{\infty} R_x(\tau) e^{-i\omega\tau} d\tau,$$

is a one dimensional spectral density. This relationship will be developed in more detail in chapter III.

CHAPTER II

DETERMINING THE PERIOD OF A PC PROCESS

In practice, one usually starts with a finite amount of discrete data. This data is generally taken with respect to time. According to Fourier analysis, a time history can be decomposed into sinusoids of different frequencies. It is this decomposition that we would like to use in order to produce an estimate of the power spectral density of a process.

The Fourier transform of discrete data can be produced by using a computer program which can be found in the library of most operating systems. The discrete Fourier transform of a vector of sampled data $\{X_n, n=0,1,\dots,N-1\}$, where N is the number of samples taken can be written as

$$\bar{X}_k = \sum_{n=0}^{N-1} X_n e^{(i2\pi kn/N)},$$

and $\{\bar{X}_k, k=0,1,\dots,N-1\}$ goes from zero frequency to $2\pi/\Delta t$ by intervals of length $2\pi/N\Delta t$ where Δt is the time between samples.

Thus, the transformed data has the same length as the original data. Also it is given with respect to frequencies which go from the zero frequency to some maximum frequency in

equal intervals. It is this characteristic of the transformed data taking values at frequencies of equal intervals which is of concern when one is trying to produce the spectrum of a PC process.

There is also a cutoff frequency

$$\omega_c = \pi/\Delta t,$$

which is the highest frequency that can be reproduced from data sampled at equal intervals Δt . This frequency is called the Nyquist frequency ω_c . This phenomenon can be understood mathematically by taking a sinusoid $X(t) = A \cos \omega_1 t$ of frequency $\omega_1 = \omega_c + \omega_2$ where $\omega_2 < \omega_c$. Then discretizing

$$\begin{aligned} X(n\Delta t) &= A \cos \omega_1 n\Delta t \\ &= A \cos(\omega_c + \omega_2)n\Delta t \\ &= A \cos(\omega_c n\Delta t + \omega_2 n\Delta t) \\ &= A \cos(\pi n + \omega_2 n\Delta t) \\ &= A(\cos \pi n \cos \omega_2 n\Delta t - \sin \pi n \sin \omega_2 n\Delta t) \\ &= A \cos(n\pi - \omega_2 n\Delta t) \\ &= A \cos(\omega_c - \omega_2)n\Delta t = A \cos \omega_a n\Delta t \end{aligned}$$

where $\omega_a = \omega_c - \omega_2$. Therefore, the frequency $\omega_1 = \omega_c + \omega_2$ is indistinguishable for sampled data from the frequency $\omega_a = \omega_c - \omega_2$. For this reason, when analyzing sampled data, \bar{X}_k , $k=0,1,\dots,N/2$ will be used corresponding to frequencies from zero to $\pi/\Delta t$.

In this thesis, the discrete data used was taken from an acoustic time history produced by a helicopter fixed with respect to the observer. The passing blades from an isolated main rotor or isolated tail rotor will produce a periodic sound pressure time history. This time history will have the

same period as that of the passing blades which produce them. The history produced by the main or tail rotor rotating along thus represents a PC process whose correlation has the same period as the time history. Recall that a periodically correlated process is a process whose correlation is a periodic function. Therefore, its correlation has a period T and a frequency f associated with it, and it is this period that one must find, first.

By definition, a function $F(x)$ is called periodic if there is a positive number T such that $F(x)=F(x+T)$ for all x . The smallest such T is called the period. The reciprocal of the period is called the fundamental frequency f_0 of the function, i.e.

$$f_0 = \frac{1}{T} .$$

Frequency corresponds to the number of cycles per second and has units in hertz. Frequency can also be viewed in terms of radian frequency ω which has units of radians per second and is related to f by

$$\omega = 2\pi f .$$

The Fourier transform of discrete data taken from a periodic time history should only have values at frequencies corresponding to the fundamental frequency and integer multiples of the fundamental frequency [2]. However, due to the nature of the transform program, values are given for the transformed data at frequencies of equal intervals. The location of these frequencies will depend on the length of the original input data to the program. Therefore, in order to

avoid having values at frequencies at which there should be zero values, data must be input to the program of lengths equal to the period or some integer multiple of the period for a PC process [2].

In this chapter, a method will be examined for determining the period of a periodically correlated process from given data. The method entails finding the lines of support for the PC process and then to use the distance between these lines to determine the period of the process.

Spectral Support

Hurd [4] introduces a technique which is useful for determining the support of a nonstationary process. The technique consists of producing the discrete Fourier transform \bar{X}_k from sampled data of a nonstationary process. Products of transforms, $\bar{X}_p \bar{X}_q$, are then plotted in the p, q plane. Subsets of these products are then summed along diagonals and normalized with respect to the main diagonal. That is, a spectral coherence is produced at coordinates (p, q) where p and q correspond to the $2\pi p/N\Delta t$ and $2\pi q/N\Delta t$ frequencies respectively. The coherence is produced by

$$|\tau(p, q, M)|^2 = \frac{\left| \sum_{m=0}^{M-1} \bar{X}_{p+m} \bar{X}_{q+m} \right|^2}{\sum_{m=0}^{M-1} |\bar{X}_{p+m}|^2 \sum_{m=0}^{M-1} |\bar{X}_{q+m}|^2}$$

This coherence is used to determine which points over the array being considered have significant values. This is

determined by choosing a threshold value and plotting points (p,q) for which the coherence exceeds the threshold. If the process is PC, the plotted points (p,q) should produce a graph (Figure 5) of dots along parallel diagonal lines. The separation between these diagonals can be used to determine whether the process is PC, and to find the period.

This technique is based on the theoretical result that the spectral support for a PC process with period T is on equidistant straight lines parallel to the main diagonal [5]. That is D_k lines where

$$D_k = \{(p,q): p=q+\frac{2\pi k}{T}, \quad k=0,\pm 1,\dots,\pm(T-1)\}.$$

Taking the case where $k=1$, the spacing of these lines is given by

$$p-q = \frac{2\pi}{T}$$

and letting $T=n\Delta t$, produces

$$p-q = \frac{2\pi}{n\Delta t}.$$

The difference $p-q$ is found from the graph. Thus the number n of time intervals Δt needed to produce the period of the process can be calculated directly. After the period has been found, our next task is to estimate the power spectral density.

CHAPTER III

A POWER SPECTRAL ESTIMATION TECHNIQUE

We will first consider the usual requirements which are placed on an estimate. For a random variable $\hat{\lambda}$ which is an estimate of an unknown parameter λ , a usual requirement is that

$$E(\hat{\lambda}) = \lambda,$$

where the expectation is over all possible values of the random variable. When this is the case, the estimate is said to be unbiased. In addition, an estimate is chosen such that its variation about its mean is as small as possible. This uncertainty of an estimate is measured by the standard deviation. That is,

$$\text{Uncertainty} = (E\{[\hat{\lambda} - E(\hat{\lambda})]^2\})^{1/2}.$$

Therefore, the usual requirements are that the estimate be unbiased and have the smallest possible uncertainty.

For stationary processes, there are two well known techniques for spectral estimation which are the Blackman - Tukey and the finite Fourier transform techniques. To produce an estimate for the power spectral density, we have chosen the finite Fourier transform technique. This method consists of taking the discrete Fourier transform of sampled

data and using the transformed data to produce a spectral estimate. Consider a stationary process $X(t)$ of which the (generalized) Fourier transform is given as

$$\bar{X}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t) e^{-i\omega t} dt$$

Since $\bar{X}(\omega)$ is complex, its autocorrelation is given as

$$E(\bar{X}(\omega_1) \bar{X}^*(\omega_2)) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 R_x(t_2 - t_1) e^{-i(\omega_1 t_1 - \omega_2 t_2)}$$

where the asterisk indicates a complex conjugate. Introducing the change of variables

$$t = \frac{t_1 + t_2}{2} \quad \text{and} \quad \tau = t_2 - t_1$$

produces

$$\begin{aligned} E(\bar{X}(\omega_1) \bar{X}^*(\omega_2)) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau R_x(\tau) e^{i(\omega_1 + \omega_2)\tau/2} \\ &\quad \times \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{-i(\omega_1 - \omega_2)t} \\ &= S_x\{(\omega_1 + \omega_2)/2\} \delta(\omega_1 - \omega_2) \end{aligned}$$

Therefore,

$$S_x(\omega) = \int_{-\infty}^{\infty} E[\bar{X}(\omega) \bar{X}^*(\omega')] d\omega'$$

In practical situations only a single sample function of length T of a random process $X(t)$ is available. Based on the above relationship just obtained, a class of power spectral estimates

$$\hat{S}_x(\omega) = W_s |\bar{X}_f(\omega)|^2$$

is introduced where

$$\bar{X}_f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} n(t) X(t) e^{-i\omega t} dt$$

is the Fourier transform of the data as seen through a data window $n(t)$. This data window is a real valued function that is zero for $t < 0$ and $t > T$, so that unavailable data are not required. And W_s is a correction factor, due to the presence of the window.

The estimate $\hat{S}_x(\omega)$ for a fixed ω is a random variable with mean

$$E\{\hat{S}_x(\omega)\} = \frac{W_s}{4\pi^2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 n(t_1) n(t_2) R_x(t_2 - t_1) e^{-i\omega(t_1 - t_2)}$$

Furthermore,

$$E\{\hat{S}_x(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{W_s}{2\pi} \int_{-\infty}^{\infty} n(t + \tau/2) n(t - \tau/2) dt R_x(\tau) e^{-i\omega\tau} d\tau$$

with $t_1 = t - \tau/2$ and $t_2 = t + \tau/2$. Here

$$u(\tau) = \frac{W_s}{2\pi} \int_{-\infty}^{\infty} n(t + \tau/2) n(t - \tau/2) dt$$

is a lag window $u(\tau)$ satisfying the following conditions:

1. $u(0) = 1$, for preserving power
2. $u(\tau) = u(-\tau)$ which makes $\hat{S}_x(\omega)$ real
3. $u(\tau) = 0$ for $|\tau| > T$

For the first condition,

$$u(0) = \frac{W_s}{2\pi} \int_{-\infty}^{\infty} n^2(t) dt = 1$$

yields

$$W_s = \frac{2}{\int_{-\infty}^{\infty} n^2(t) dt}.$$

Therefore, the estimate becomes

$$\hat{S}_x(\omega) = \frac{2\pi |X_f(\omega)|}{\int_{-\infty}^{\infty} n^2(t) dt}$$

The second condition, $u(\tau)$ being even, is obviously satisfied while the third condition is also satisfied since $u(t)$ is the convolution of two data windows that are only nonzero in the range $(0, T)$.

The estimate developed above are equivalent in expectation to a class of estimates developed by Blackman and Tukey in which

$$\hat{S}_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(\tau) \hat{R}_x(\tau) e^{-i\omega\tau} d\tau$$

where $\hat{R}_x(\tau)$ is an estimate of the autocorrelation function of the process and $u(\tau)$ is a lag window as described above. The expectation of this estimate is

$$E(\hat{S}_x(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(\tau) R_x(\tau) e^{-i\omega\tau} d\tau.$$

This mean spectral estimate can be shown to equal the convolution of the actual spectral density with a "spectral window" which is merely the Fourier transform of the lag window. Since the autocorrelation is an even function, it can be seen that

$$\begin{aligned}
E(\hat{S}_x(\omega)) &= \frac{1}{2\pi} \int_0^{\infty} u(\tau) R_x(\tau) \cos \omega \tau d\tau \\
&= S_x(\omega) - \frac{1}{2\pi} \int_T^{\infty} R_x(\tau) \cos \omega \tau d\tau
\end{aligned}$$

for the case of a "boxcar" lag window, $u(\tau)=1$. Therefore, the estimate for the power spectral density is biased, but it becomes unbiased as $T \rightarrow \infty$. In a similar fashion to the first technique, we suggest the following technique for estimating the two dimensional power spectral density of a nonstationary process. The power spectral density for a nonstationary process can be written as

$$\begin{aligned}
S_x(\omega_1, \omega_2) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 R_x(t_1, t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)} \\
&= E\{\bar{X}(\omega_1) \bar{X}^*(\omega_2)\},
\end{aligned}$$

where

$$\bar{X}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t) e^{-i\omega t} dt.$$

Here $S_x(\omega_1, \omega_2)$ can not be simplified to $S_x(\omega)$, since the autocorrelation $R_x(t_1, t_2)$ depends on both variables t_1 and t_2 .

Therefore, from a sample function of length T of a nonstationary process $X(t)$, a class of power spectral estimates

$$\hat{S}_x(\omega_1, \omega_2) = W_s \bar{X}_f(\omega_1) \bar{X}_f^*(\omega_2)$$

can be introduced where

$$\bar{X}_f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} n(t) X(t) e^{-i\omega t} dt$$

is the Fourier transform of the data as seen through a window function $n(t)$ as in the stationary case.

The mean of the estimate is

$$E(\hat{S}_x(\omega_1, \omega_2)) = \frac{W_s}{4\pi^2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 n(t_1)n(t_2)R_x(t_1, t_2)e^{-i(\omega_1 t_1 - \omega_2 t_2)}$$

The above analysis for the estimate being biased on $\omega_1 = \omega_2$ in the stationary case can be seen to hold when ω_1 is not equal to ω_2 .

It can be shown [2], that the spectral estimate of the finite Fourier transform approach, in the stationary case, is essentially a chi-square random variable with two degrees of freedom. Therefore, variability of the estimate can be reduced by breaking the estimate into N_B blocks of length T_B such that $N_B T_B = T$. A spectral estimate $\hat{S}_{xj}(\omega)$ for $j=1, 2, \dots, N_B$ will then be taken over each block. If the blocks are assumed to be independent, the average of the block estimates

$$\bar{S}_x(\omega) = \frac{1}{N_B} \sum_{j=1}^{N_B} \hat{S}_{xj}(\omega)$$

is essentially a chi-square random variable with $K=2N_B$ degrees of freedom. It can be shown with a graph of the variation of a chi-square random variable (Figure 2) where $a(k)$ is the left bound and $b(k)$ is the right bound that

$$P\{\bar{S}_x(\omega)/a(k) > S_x(\omega) > \bar{S}_x(\omega)/b(k)\} = 0.80 .$$

Therefore, 80 percent of the time the actual spectral density will lie between $1/a(k)$ and $1/b(k)$ times the spectral

estimate.

The variability of such estimates is intimately linked to their resolution. Full resolution refers to two sinusoids of the same amplitude being completely distinguishable when viewed through the spectral window function in the frequency domain. For the finite Fourier transform technique, full resolution requires that frequencies be roughly separated by

$$\Delta\omega = 2\pi/T \text{ or } \Delta f = 1/T.$$

Now by breaking the data into N_B blocks, the effective length has changed from T to T_B . Thus, the bandwidth of the estimate decreases to

$$\Delta f = 1/T_B$$

Since, $K=2N_B=2T/T_B$

$$K=2\Delta f T$$

which shows the tradeoff between variability and frequency resolution.

The estimate in the stationary case

$$\hat{S}_x(\omega) = W_s \bar{X}_f^*(\omega) \bar{X}_f(\omega)$$

is essentially a chi-square random variable. Although it is not a chi-square random variable, the estimate for the PC process

$$\hat{S}_x(\omega_1, \omega_2) = W_s \bar{X}_f^*(\omega_1) \bar{X}_f(\omega_2)$$

similarly reduces in variability, when the process is blocked in integer multiples n of the period. That is,

$$T_B = np$$

where p is the period of the PC process.

CHAPTER IV

SPECTRAL ANALYSIS OF HELICOPTER NOISE

We will first examine the correlation of the acoustic pressure time history X_n from an isolated helicopter rotor. X_n represents a discrete process which reflects the fact that the time history is sampled at discrete intervals. It is this sampled data which is used to produce a spectral estimate. This time history is doubly periodically correlated (DPC). That is

$$R(m, n) = EX_m X_n^*$$

is periodic in both m and n . Therefore, its correlation function being periodic in m can be written as

$$(*) \quad R(m, n) = \sum_{k=0}^{T-1} R_k(n) e^{2\pi i m k / T}, \quad \text{for all } m$$

And since $R(m, n)$ is also periodic in n , we can write

$$R(m, n+T) = R(m, n).$$

Therefore,

$$\sum_{k=0}^{T-1} R_k(n) e^{2\pi i m k / T} = \sum_{k=0}^{T-1} R_k(n+T) e^{2\pi i m k / T}, \quad \text{for all } m, n$$

For a fixed n we have

$$\sum_{k=0}^{T-1} [R_k(n+T) - R_k(n)] e^{2\pi i m k / T} = 0, \quad \text{for all } m$$

which implies that

$$R_k(n+T) = R_k(n) , \quad \text{for all } k$$

And since n was arbitrary, $R_k(n)$ is periodic for each k .

Therefore, we can write

$$R_k(n) = \sum_{j=0}^{T-1} a_{kj} e^{-2\pi nj/T}$$

Substituting into (*) we get

$$R(m,n) = \sum_{k=0}^{T-1} \left[\sum_{j=0}^{T-1} a_{kj} e^{-2\pi inj/T} \right] e^{2\pi imk/T}.$$

Hence

$$R(m,n) = \sum_{k=0}^{T-1} \sum_{j=0}^{T-1} a_{kj} e^{2\pi i(mk+nj)/T}$$

This means that the spectrum of a DPC process is supported on T^2 points $\{(2\pi k/T, 2\pi j/T) : j, k = 0, 1, \dots, T-1\}$ (Figure 3).

Now we can try to get a representation for the process X_n . We must first utilize the fact that the matrix a_{kj} is positive definite. To do this we must show that

$$DAD^* = \sum_{k,j} d_k a_{kj} d_j \geq 0$$

for each vector $D = (d_0, d_1, \dots, d_{T-1}) \in C^T$. Let $(c_0, c_1, \dots, c_{T-1})$ be a vector in C^T such that

$$\sum_{n=0}^{T-1} c_n e^{2\pi ink/T} = d_k, \quad \text{for every } k=0, 1, \dots, T-1.$$

This is possible by Fourier theory. Now we can write

$$\begin{aligned} \sum_{k,j=0}^{T-1} d_k a_{kj} d_j^* &= \sum_{k,j=0}^{T-1} \left(\sum_{m=0}^{T-1} c_m e^{2\pi imk/T} \right) a_{kj} \left(\sum_{n=0}^{T-1} c_n^* e^{2\pi inj/T} \right)^* \\ &= \sum_{m=0}^{T-1} \sum_{n=0}^{T-1} c_m \left(\sum_{k,j=0}^{T-1} a_{kj} e^{2\pi i(mk-nj)/T} \right) c_n^* \end{aligned}$$

$$\begin{aligned}
&= \sum_{m=0}^{T-1} \sum_{n=0}^{T-1} c_m R(m,n) c_n^* \\
&= E \left\{ \left(\sum_{m=0}^{T-1} c_m X_m \right) \left(\sum_{n=0}^{T-1} c_n^* X_n^* \right) \right\} \\
&= \left\| \sum_{n=0}^{T-1} c_n X_n \right\|^2 \geq 0 \quad \text{Q.E.D.}
\end{aligned}$$

Since a_{kj} has been shown to be positive definite, we know there is a vector $(y_0, y_1, \dots, y_{T-1})$ of Gaussian random variables with mean $(0, 0, \dots, 0)$ and covariance matrix (a_{kj}) . So

$$a_{kj} = E(y_k y_j^*) \quad \text{for every } k, j = 0, 1, \dots, T-1.$$

Therefore, we can write (4.2) as

$$\begin{aligned}
R_x(m,n) &= \sum_{k=0}^{T-1} \sum_{j=0}^{T-1} E(y_k y_j^*) e^{2\pi i(km-jn)/T} \\
&= E \left\{ \left(\sum_{k=0}^{T-1} e^{2\pi i k m / T} y_k \right) \left(\sum_{j=0}^{T-1} e^{2\pi i j n / T} y_j^* \right) \right\}
\end{aligned}$$

This means that

$$X_n = \sum_{k=0}^{T-1} e^{2\pi i k n / T} y_k, \quad \text{for every } n=0, \pm 1, \pm 2, \dots$$

or

$$X_n = y_0 + y_1 e^{2\pi i n / T} + y_2 e^{(2)2\pi i n / T} + \dots + y_{T-1} e^{(T-1)2\pi i n / T}.$$

Therefore,

$$X_n = X_{n+T}.$$

And since X_n is periodic, prediction is easy once the period of the process is known.

The object of this thesis has been to develop a technique for analyzing helicopter noise in a more exact way. The

standard approach is to treat the data as if it were stationary and to produce a one dimensional spectrum from which the data will be analyzed. In actuality the data produced from the isolated tail rotor and isolated main rotor is periodically correlated which requires a two-dimensional spectrum to completely represent the spectrum of the data. Further the data from the main and tail rotor combination is basically the sum of two incommensurate periodic components which means that this data is neither periodic nor stationary. Hence, this data also requires a two dimensional spectrum.

In trying to develop a technique to analyze helicopter noise the necessity for the spectrum to be two-dimensional is utilized. As discussed in chapter 3, an estimate was developed for the power spectrum of a nonstationary process. And it was this estimate that was used to produce two dimensional spectral estimates from the data. The technique consists of estimating the spectrum over a region of values for ω_1 and ω_2 and then block averaging the estimate to reduce variability. In the case of the isolated main and tail rotors blocking was done as a multiple of the period of process. This was done because of the characteristics of a periodic process. A periodic process has an amplitude spectrum as discussed above with support only on lattice points. And to insure that these points are actually viewed, a length of data which is a multiple of the period of the process must be used when the data is Fourier transformed.

The data used to produce the spectral estimates in this thesis was taken from a wind tunnel test conducted at NASA/Langley. The Sikorsky Aircraft's Basic Model Test Rig was used in the experiment details of which can be found in [6]. The data was taken from several locations around the helicopter model (see Figure 4). By analyzing data from several different positions, the noise pattern produced could be used to determine how the noise is radiated and in which direction most of the noise travels.

In producing the two dimensional spectrum for the isolated tail rotor, program 1 (see Appendix A) was created to produce values of the estimate. The tail rotor had a blade passage frequency of around 450 Hz which was determined by graphed data (see Figure 5, lines for the shaft frequency are also present) produced by computer program 2 (see Appendix B). The motivation for this program was discussed in chapter II. The blade passage frequency was also verified analytically using the given test conditions. The passing blades produce a corresponding periodic acoustic pressure time history with a fundamental frequency of 450 Hz. It is this time history from which the sampled data was taken. After the data was input to program 1 (Appendix A), a set of values were given in the output for the spectral estimate at frequencies ω_1 and ω_2 going from zero to a chosen upper frequency (for us 4800 Hz). We viewed the data and decided that due to the background noise at low frequencies the scaling of the graph

did not allow enough of the power at the fundamental frequency and at harmonics of the tail rotor period to be shown. Therefore, we decided to filter the data using a Chebyshev digital filter. All frequencies below 250 Hz were removed, and a graph of the output data was viewed once again. This time more detail of the power at the fundamental frequencies and its harmonics could be seen (see Figures 6,7,8,9).

The graphs show most of the power for the tail rotor at the fundamental frequency (450 Hz) and the first and second harmonics (900 and 1350 Hz respectively). There are also components off the main diagonal which represent the correlation of the amplitude of sinusoids at frequencies ω_1 and ω_2 where $\omega_1 \neq \omega_2$.

As for the data sampled from the isolated main rotor time history, this data is periodic with a frequency of 95 Hz. The first graphs of this data showed high power levels at around 95 Hz. Since the background noise was also high around 95 Hz, we decided to filter frequencies below 250 Hz out of this data also. This resulted in the second harmonic of the main rotor noise (285 Hz) being dominant. At microphone 3 there is noticeable power at the third and fourth harmonics also. At microphone 5 the background tonal noise between 800 and 1300 Hz as reported in [6] is apparent. See figures 10, 11, 12 and 13 for the spectral estimates produced from data taken by microphones 2 through 5.

The spectrum of the combined noise was of interest due

to the fact that if noise for the isolated main and tail rotors is independent, the theory shows that the spectrum for the combined noise is just the sum of the spectra of the isolated main and tail rotors. After filtering the low frequencies as above, it could be seen from figures 14, 15, 16 and 17 that to some degree the combined noise had a spectrum which was the sum of the main and tail rotor spectra at corresponding microphones. However, the question was raised as to the independence of the noise from the main and tail rotors when both rotors are operating due to the possibility of the main rotor wake being swept back into the tail rotor.

We now took another approach. We took the data from the main rotor and added it directly to the data from the tail rotor. Then we produced the two dimensional spectrum (see Figures 18, 19, 20 and 21) for this added noise. As the theory shows, these spectral estimates are the sum of the spectra of the isolated main and tail rotors at corresponding microphones. After comparing spectrums of the added noise to those of the combined noise, we concluded that there is some degree of dependence between the noise from the main rotor and tail rotor when the combined noise is being produced.

It should be noted again that the combined noise is not periodically correlated. It is the sum of two periodically correlated processes with incommensurate periods. According to theory, if the two periodically correlated processes are independent, the resulting spectrum should equal the sum of

the spectra of each process. Since the combined noise can only be blocked with respect to one period, we chose to block the data with respect to the period of the tail rotor noise. This will naturally result in the estimated spectrum being skewed to some degree. However, the same amount of skewing occurs in the estimated spectrum for the added noise which is also blocked with respect to the tail rotor noise. Therefore, the estimated spectrum of the combined and added noise can be viewed equally with respect to theory.

A. Assumed Stationarity

The standard method for handling data taken from helicopter noise is to treat the data as if it were stationary. This results in a one dimensional spectrum along the $\omega_1 = \omega_2$ diagonal. Therefore, the spectrum excludes all information concerning correlation of the sinusoidal amplitudes at frequencies where $\omega_1 \neq \omega_2$. Although these correlations need further investigation, they can not be neglected if the spectrum of a nonstationary process is to be completely studied. Examples of sound pressure spectra, for the tail rotor produced using this method, are given in figures 22, 23, 24 and 25. The graphs produced contain 22 degrees of freedom and a frequency resolution of 49 Hz. The noticeable peaks are the fundamental frequencies and subsequent harmonics. In the graphs the pressure level drops off very rapidly. Notable peaks start to drop off after 3000 Hz. Due to the frequency range of audible sound (20 hz to 20

khz), the ear suggests high tonal levels above 3000 Hz also. However, this is not evident with this technique.

B. Phase Shifted Data

A periodically correlated process will become stationary by applying a random phase shift which is uniformly distributed over the period of the process [7]. This stationary process can then be adequately analyzed by the standard method discussed above. This new stationary process which has been produced is no longer the original process. And thus, no longer contains information about the correlation of Fourier components at different frequencies, Hardin and Miamee [3]. To implement the technique, we take a length of data and break it into blocks. Each block is of length necessary to obtain a desired frequency resolution plus the period of the underlying process. When the program is implemented, a random function call chooses a sample index uniformly distributed over the period of the process. Starting with this value, the program Fourier transforms enough samples to produce the desired frequency resolution. This is done for each block of data (see program 3 Appendix C). This technique results in wastage of sample values from the start of the record up to the chosen index, but does implement the random phase shift while maintaining the desired number of samples per block.

The sound pressure spectra produced from this shifted data can be compared to that of the unshifted data one

dimensional spectra. After viewing the graphs in figures 26, 27, 28 and 29, it is apparent that there is significant power at frequencies above 3000 Hz. The impression which we get when overlaying graphs of the shifted and unshifted data is that by not shifting the data, we get a smoothed spectrum as if a moving average had been applied to the spectrum. There are additional peaks in the spectrum which appear to correspond to the shaft frequency. Since the shaft frequency is one fourth of the blade passage frequency, there could be three peaks between the harmonics of the spectrum for the tail rotor noise. This is evident in [6] in which a frequency resolution of 12 Hz is used. This resolution is fine enough to show the shaft harmonics. However, our graphs have a frequency resolution of 50 Hz which is not fine enough to completely show the shaft harmonics. This technique of shifting the data appears to be useful in resolving harmonics of the transformed data, and also rests on a firm theoretical foundation.

The output resulting from this method was also used in order to produce a two dimensional spectral estimate. The graphs for the data taken at microphones 2 through 5 are given in figures 30, 31, 32 and 33. The graphs contain off diagonal values which are not to be expected for stationary data. We suspect that this is due to our limited data length. The time history could only be broken into eleven blocks in which the data is shifted. This small amount of data does not suffice

for a uniform representation over the period of the process, which is required in order to produce a stationary process from a PC process.

CONCLUSION AND RECOMMENDATIONS

In conclusion we feel that we have successfully developed a method for producing a two dimensional spectral estimate. This estimate is known to contain more information than the one dimensional estimate which results from considering nonstationary data as being stationary. We have shown that applying a random shift to a periodically correlated process results in a more useful spectrum for viewing the higher harmonics when the data is analyzed as stationary. The harmonics resulting from the standard method discussed above appear to drop off too quickly.

For further study, we recommend that values off the main diagonal of the spectral estimate be investigated as to how their presence can be used to characterize the process itself.

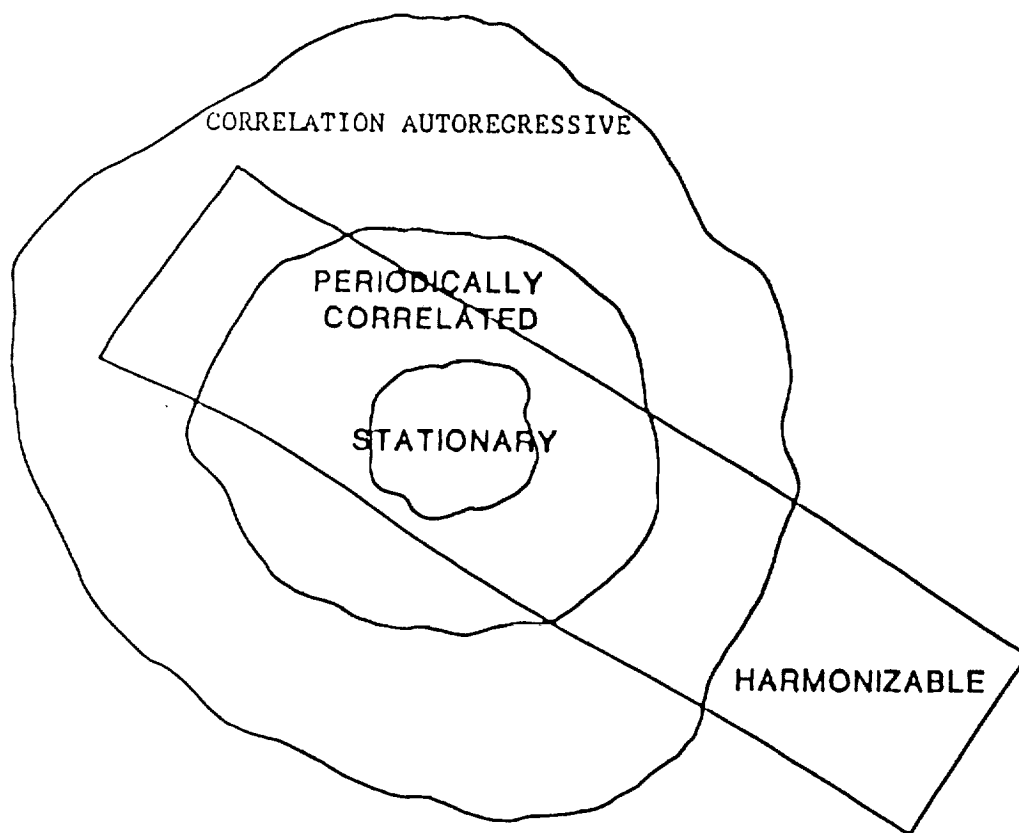


Figure 1. Venn Diagram of Class Relations

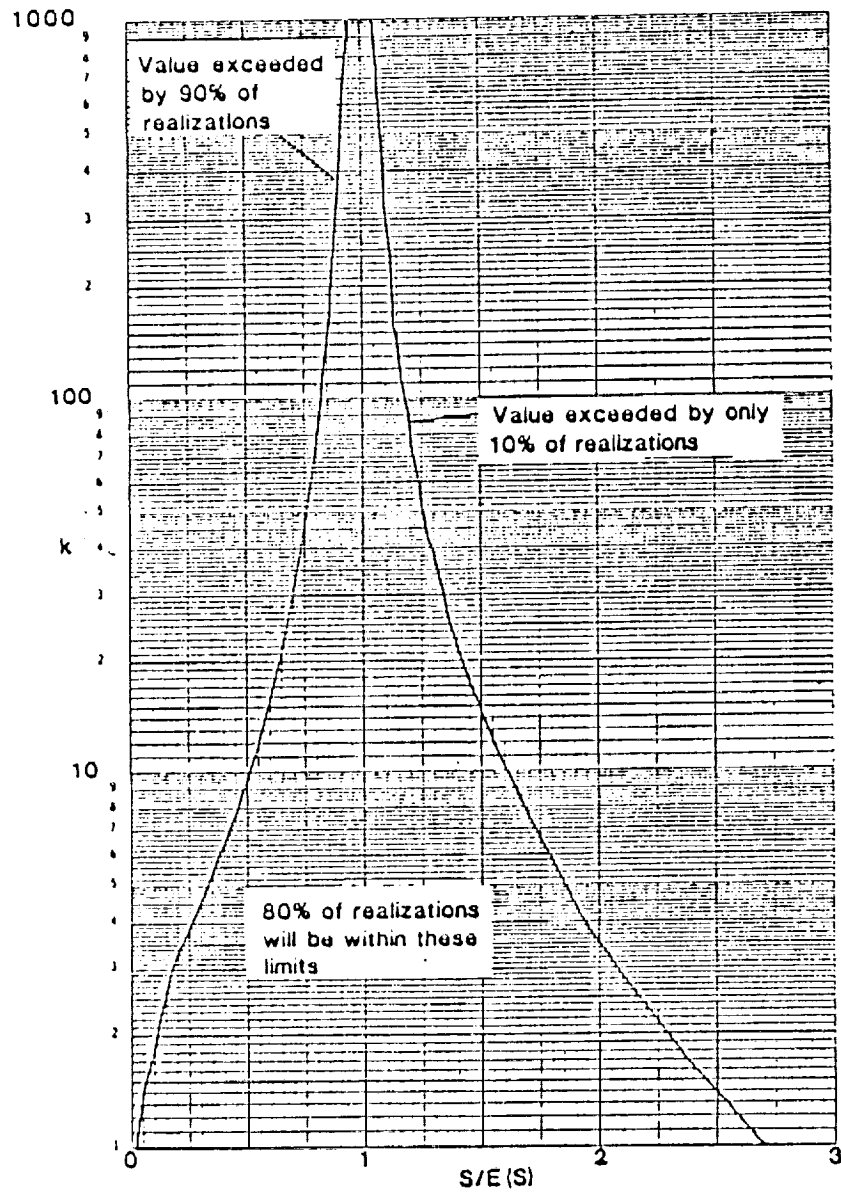


Figure 2. Variation of Chi-square Random Variable

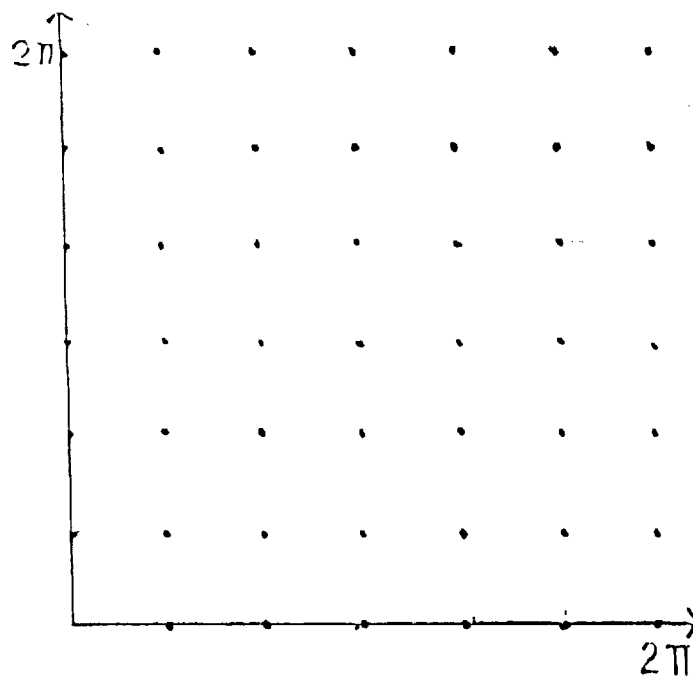


Figure 3. Spectral Support for DPC Processes

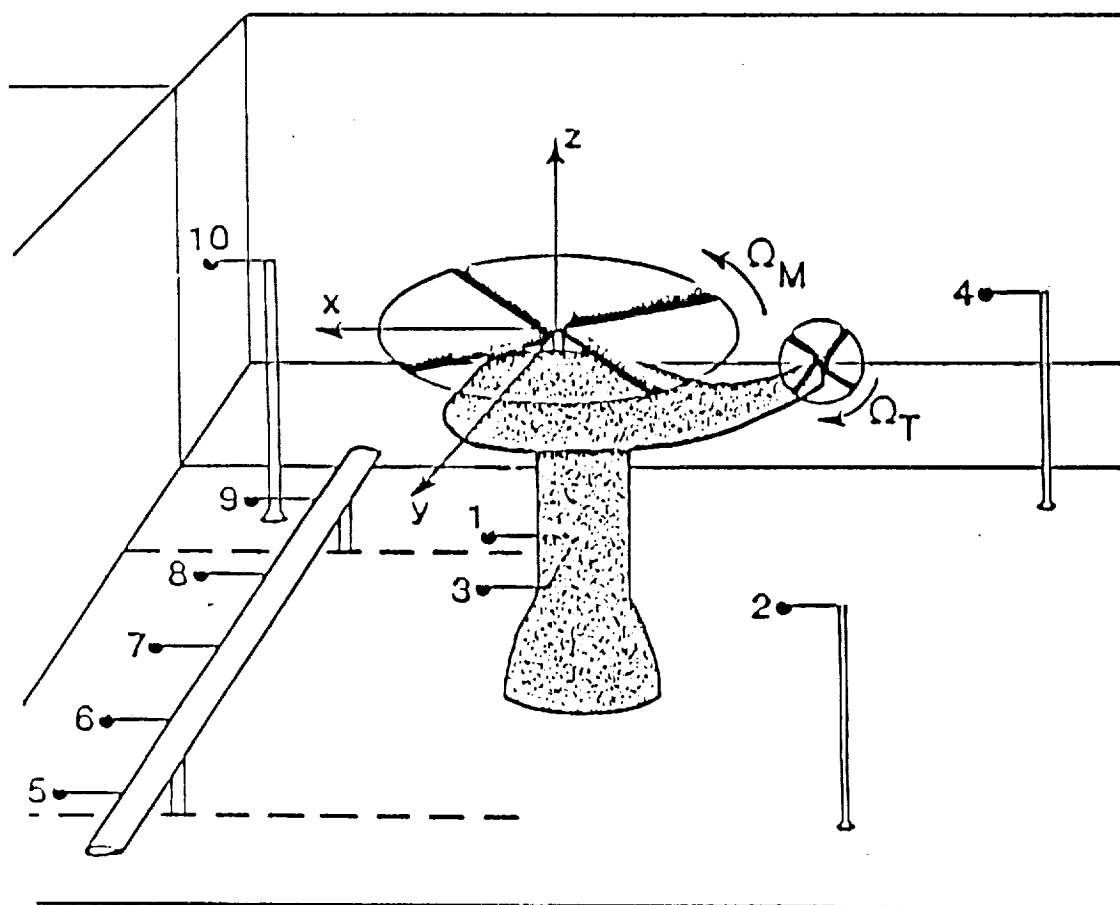


Figure 4. Diagram of Microphone Locations

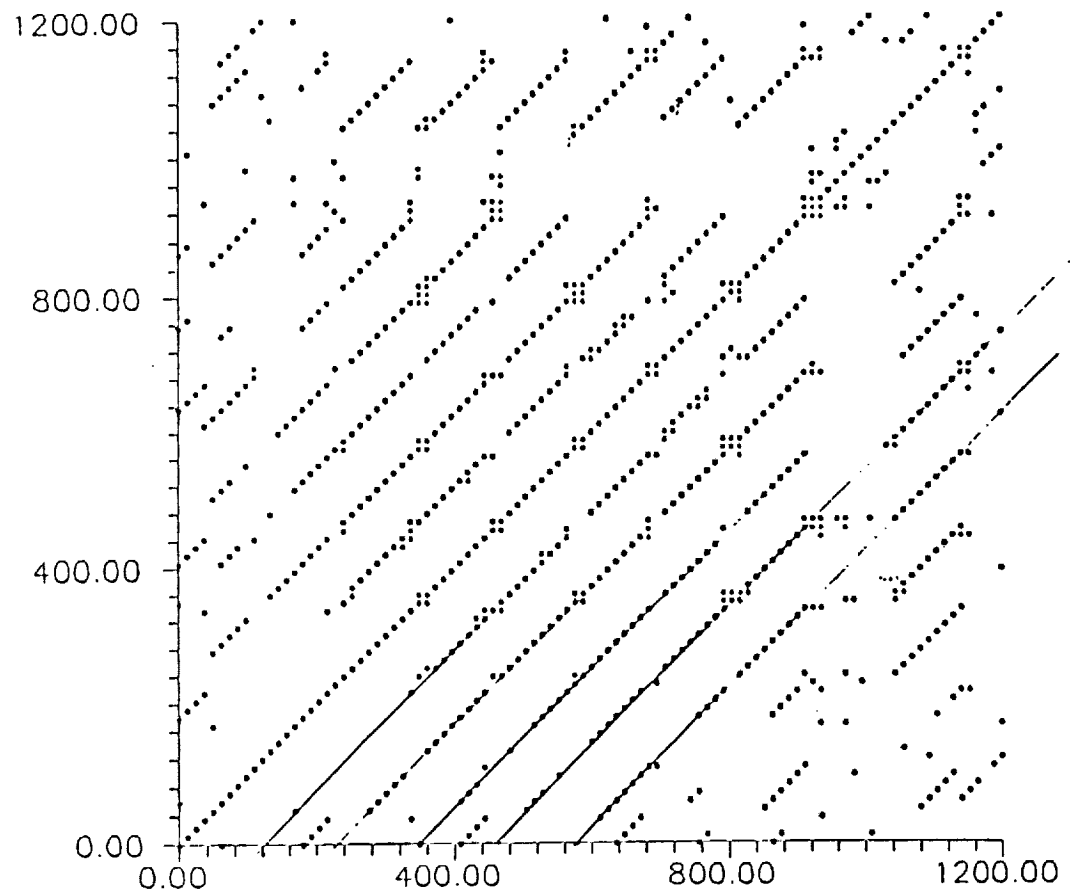


Figure 5. Spectral Coherence for Isolated Tail Rotor Data

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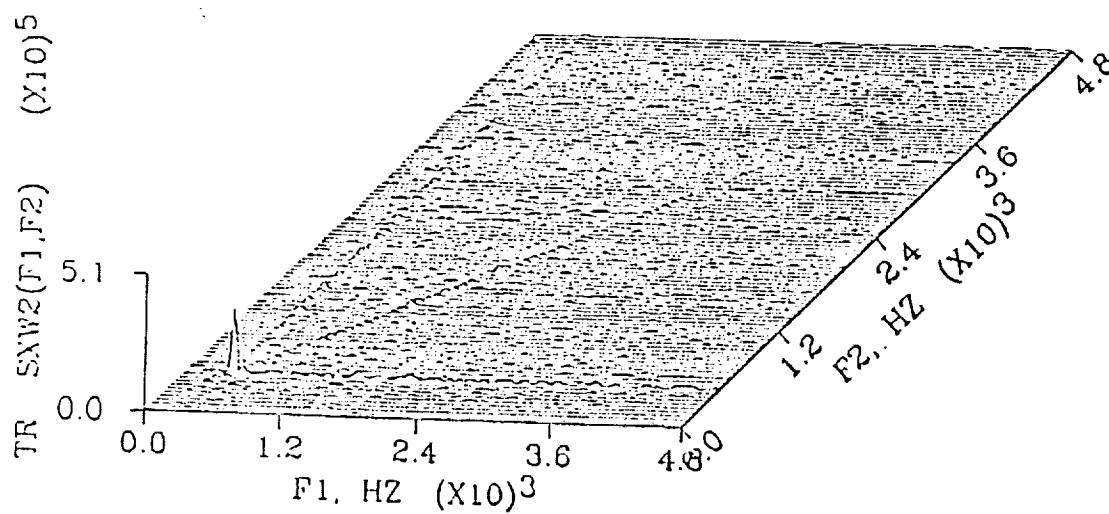


Figure 6. Tail Rotor Spectrum Microphone 2

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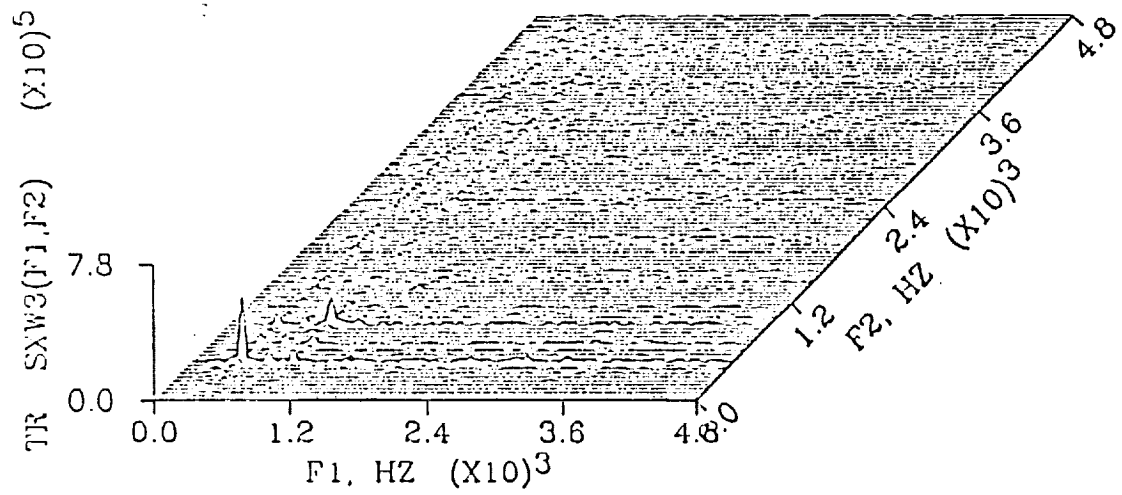


Figure 7. Tail Rotor Spectrum Microphone 3

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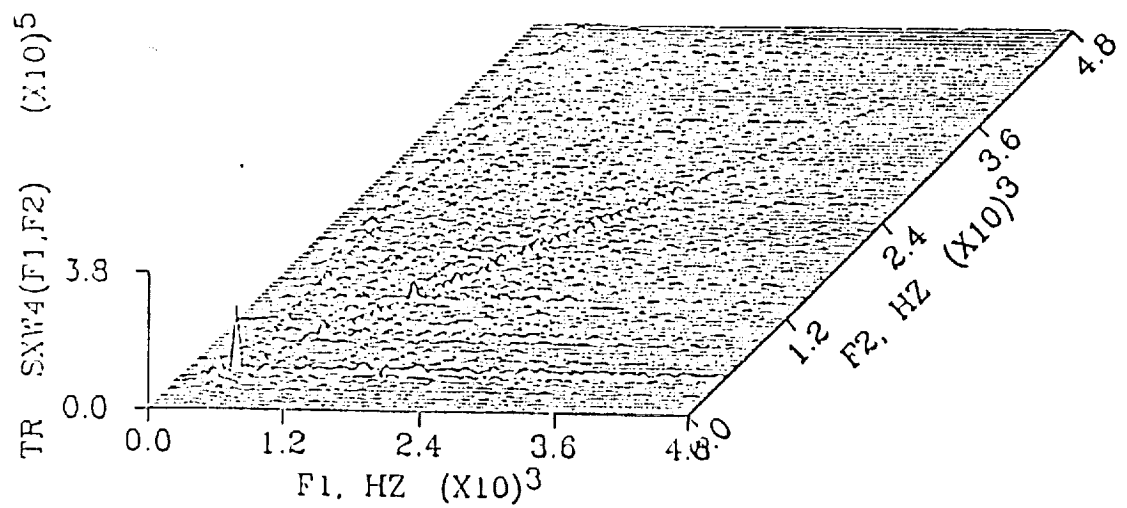


Figure 8. Tail Rotor Spectrum Microphone 4

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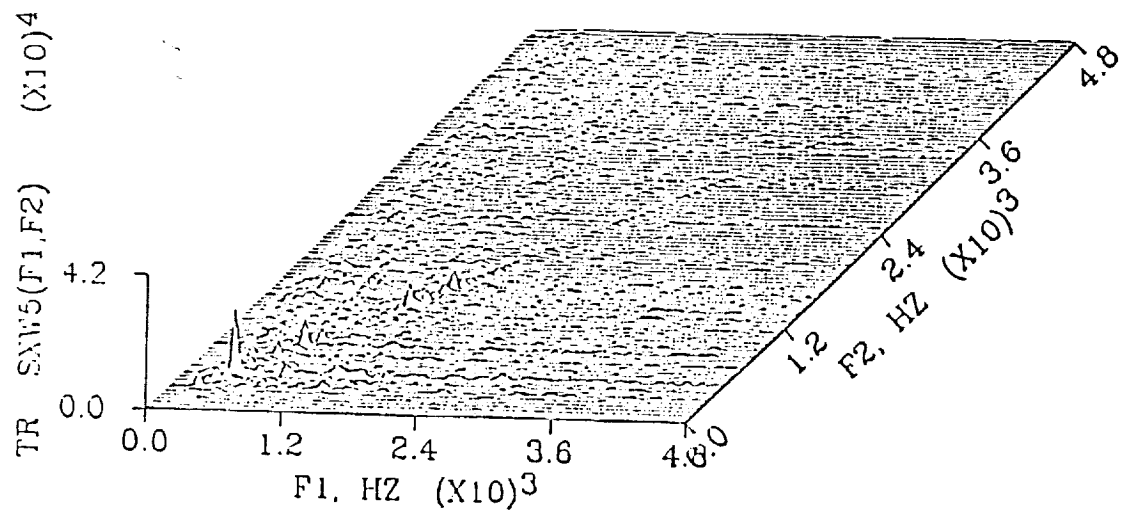


Figure 9. Tail Rotor Spectrum Microphone 5

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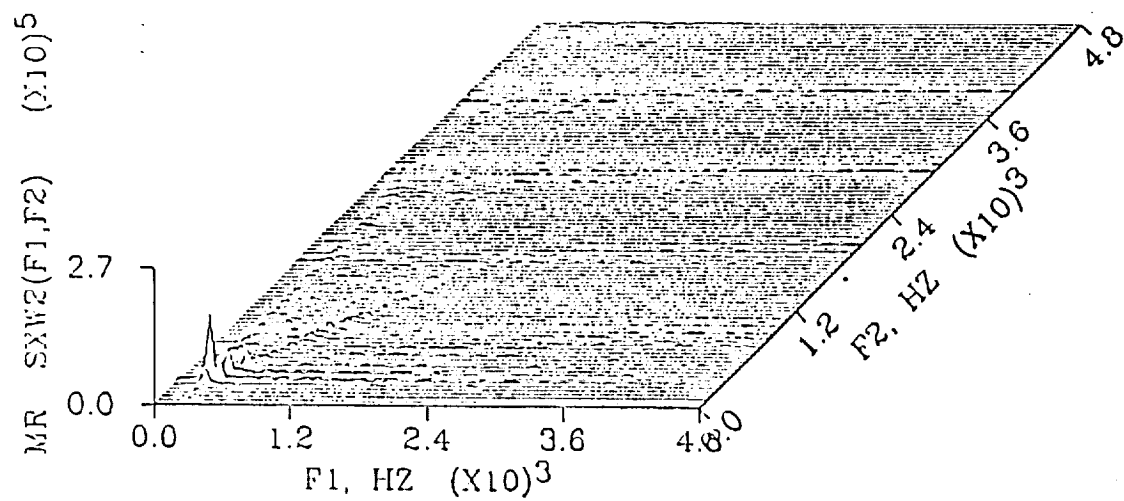


Figure 10. Main Rotor Spectrum Microphone 2

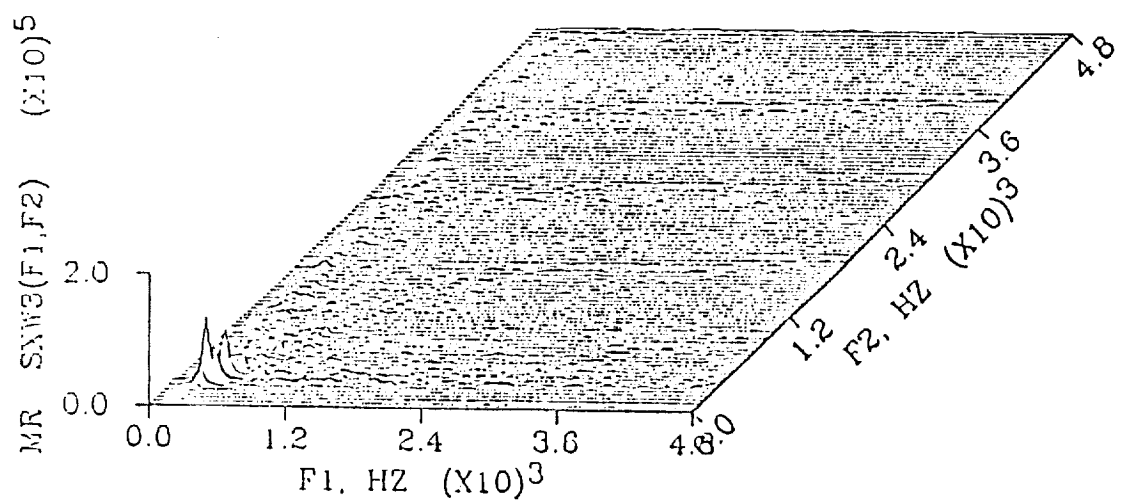


Figure 11. Main Rotor Spectrum Microphone 3

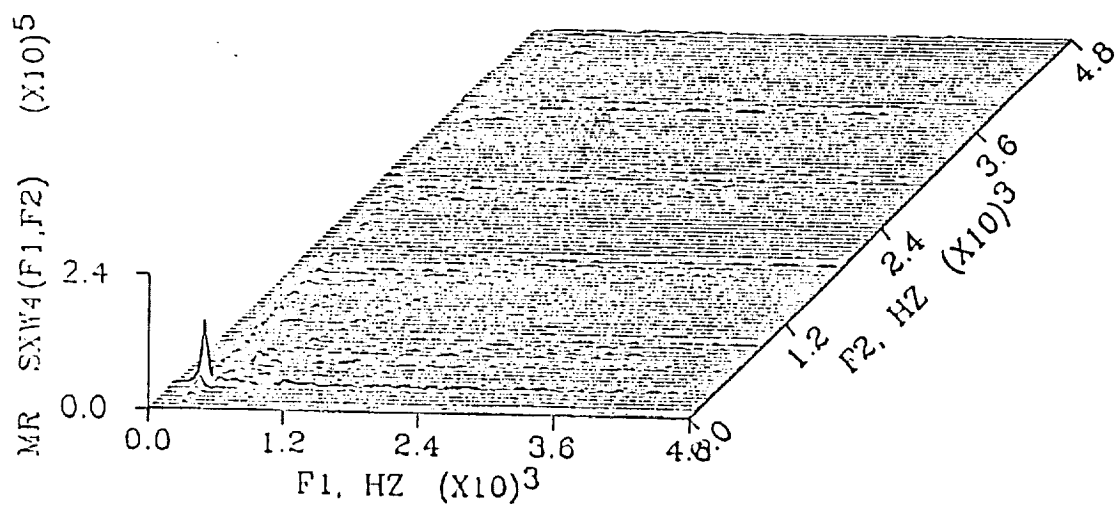


Figure 12. Main Rotor Spectrum Microphone 4

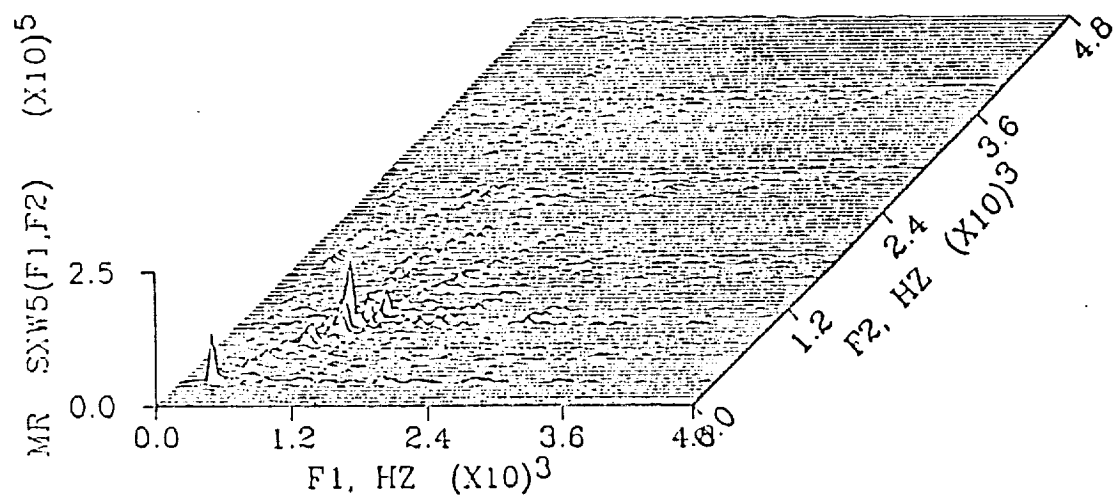


Figure 13. Main Rotor Spectrum Microphone 5

(X10)⁵

SXW2(F1,F2)

Combined M & TR

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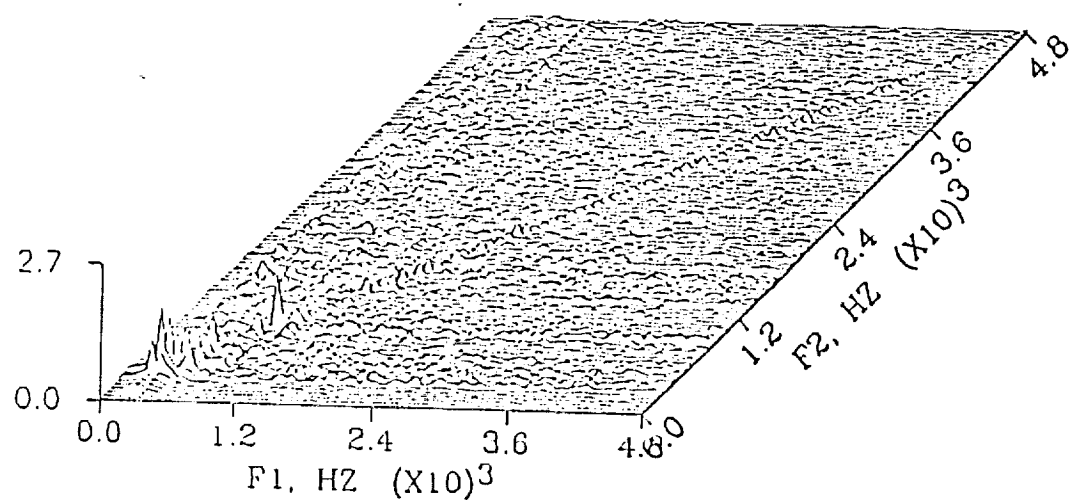


Figure 14. Combined Spectrum Microphone 2

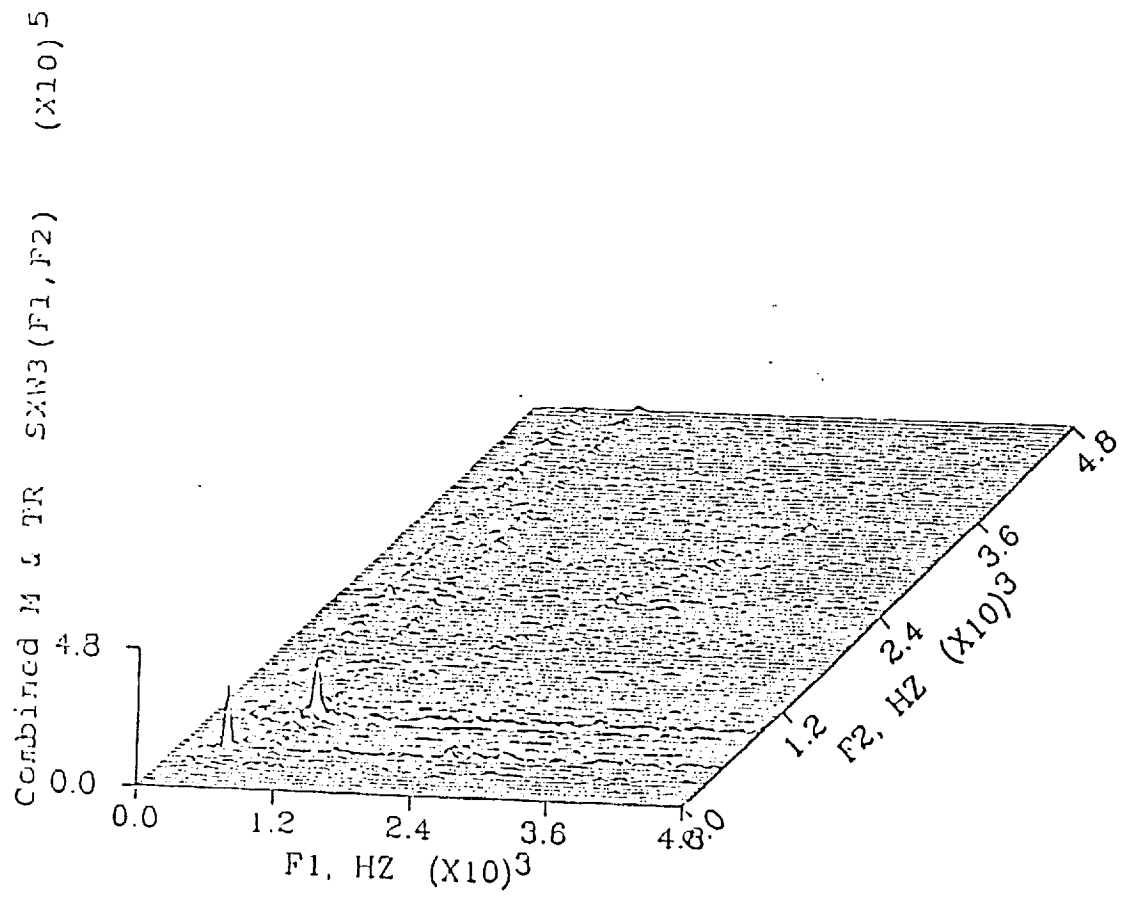


Figure 15. Combined Spectrum Microphone 3

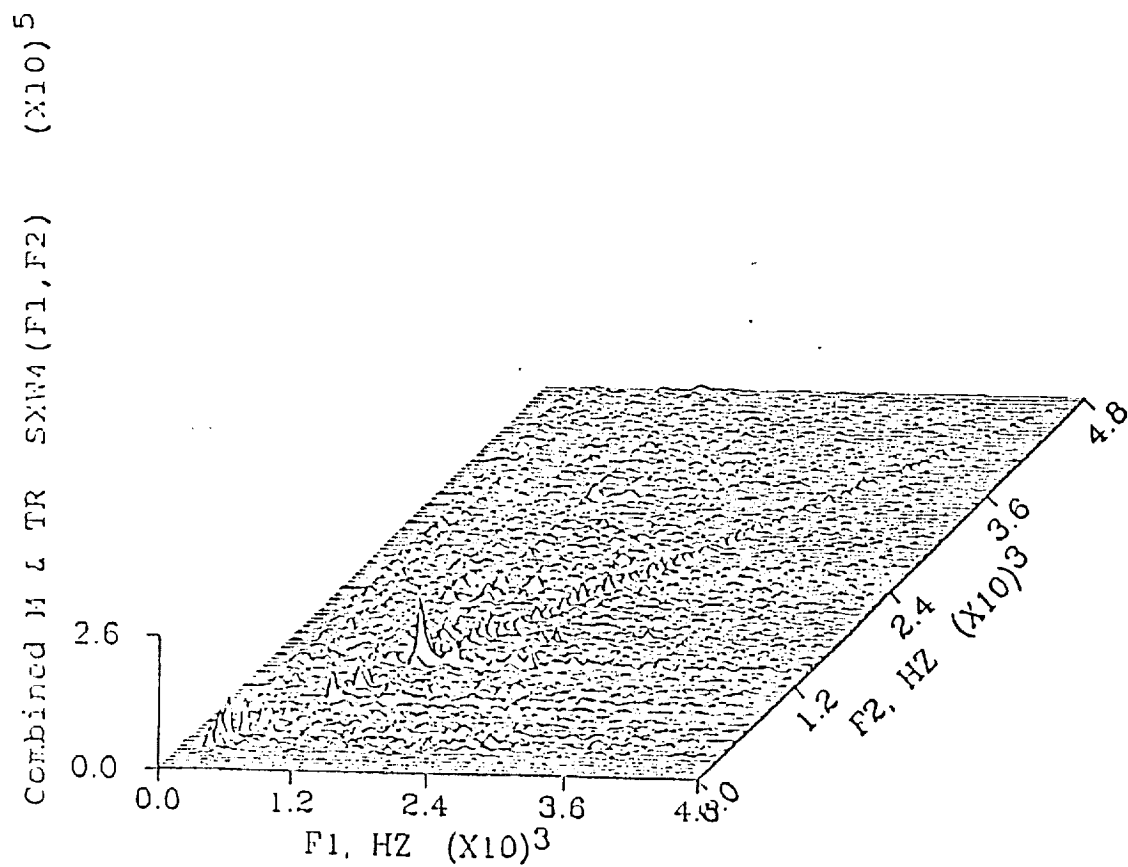


Figure 16. Combined Spectrum Microphone 4

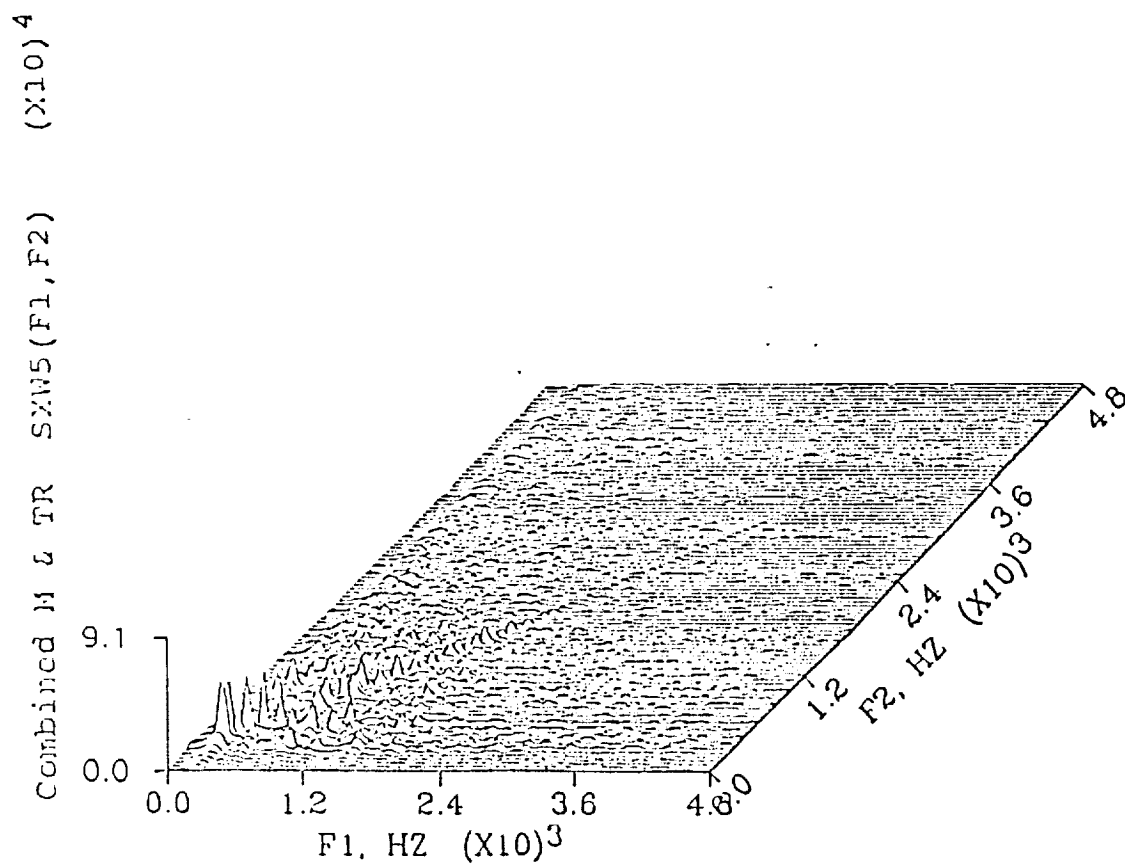


Figure 17. Combined Spectrum Microphone 5

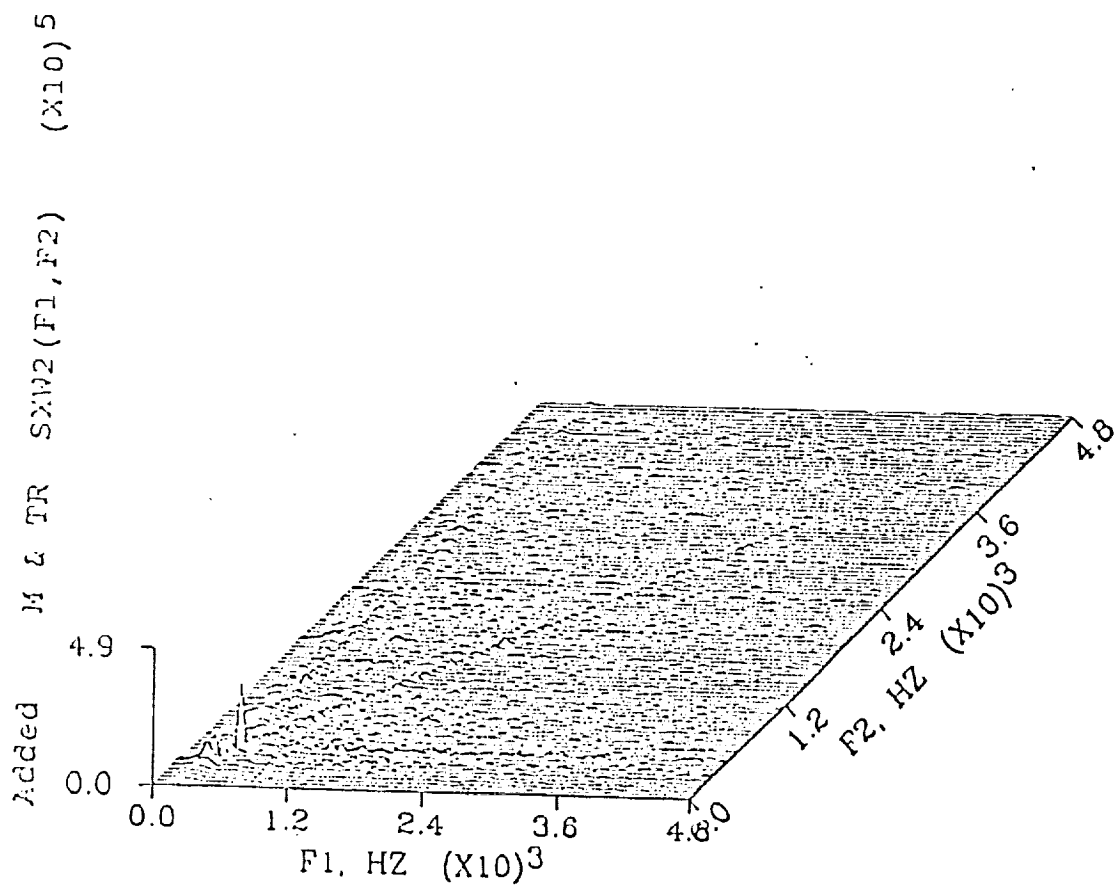


Figure 18. Added Spectrum Microphone 2

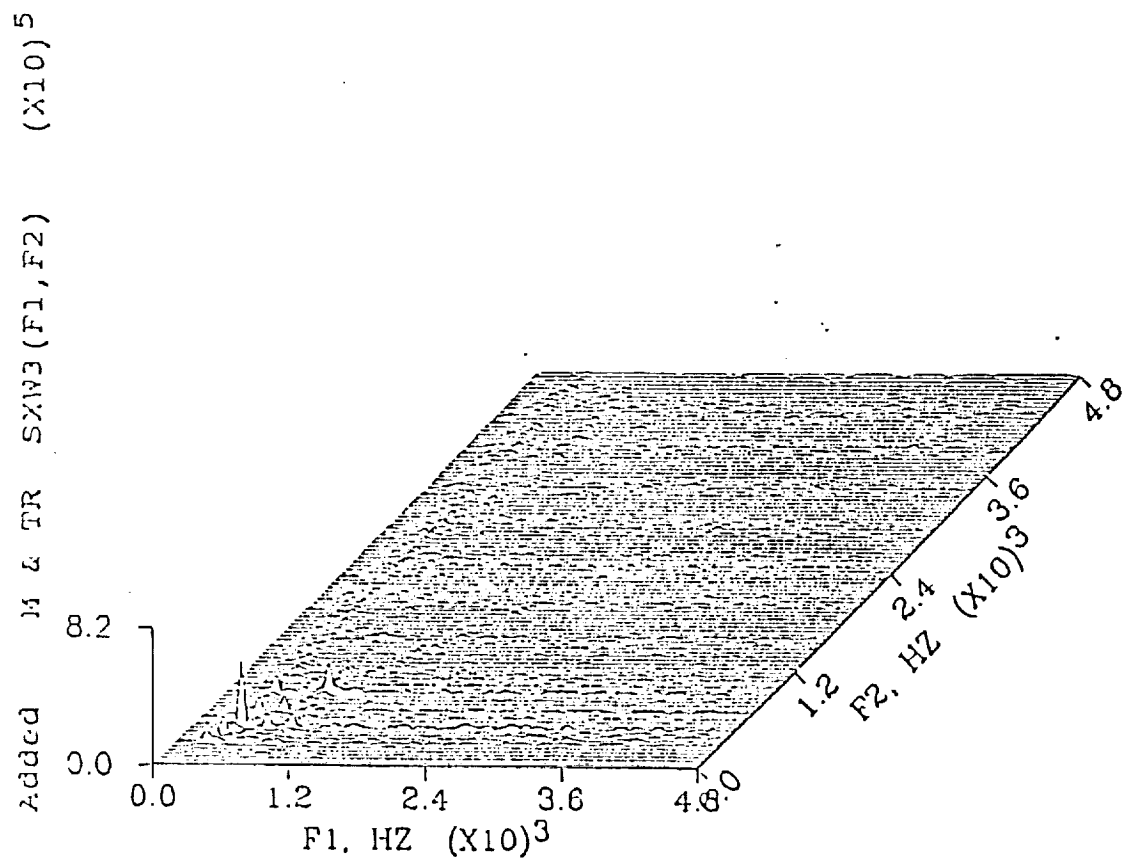


Figure 19. Added Spectrum Microphone 3

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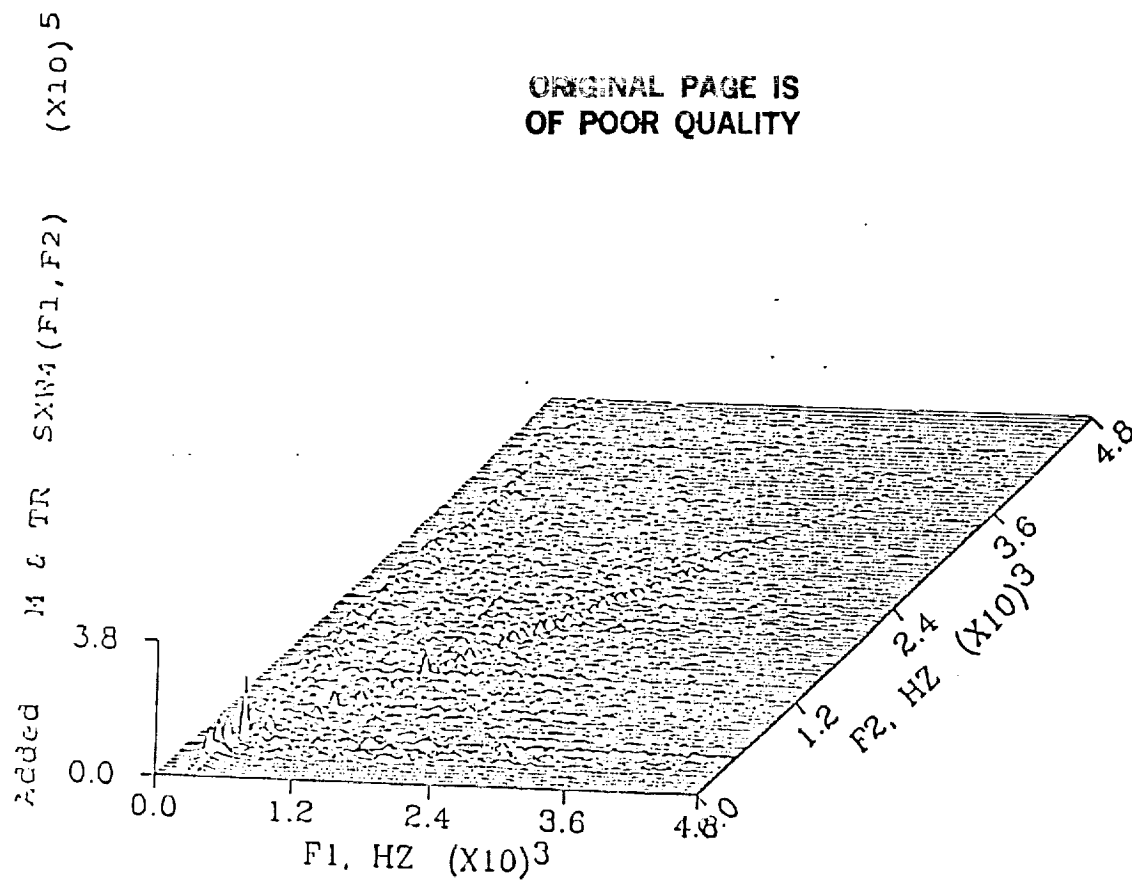


Figure 20. Added Spectrum Microphone 4

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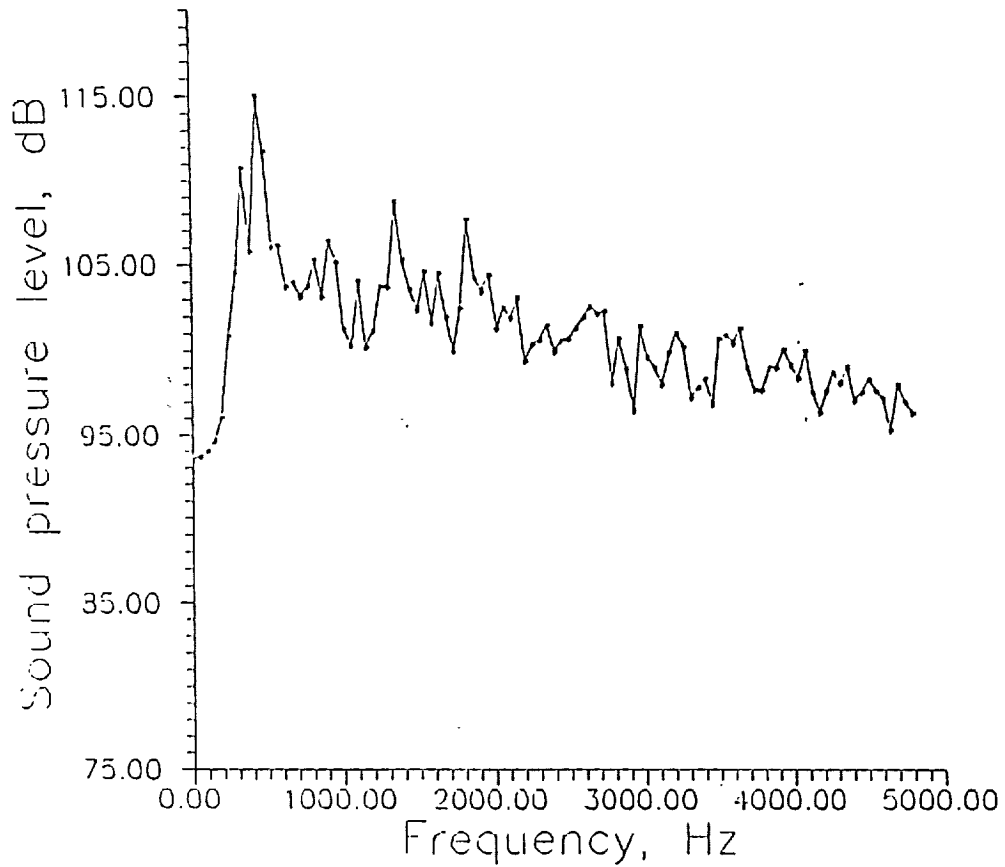


Figure 22. Tail Rotor Sound Pressure Level
Microphone 2

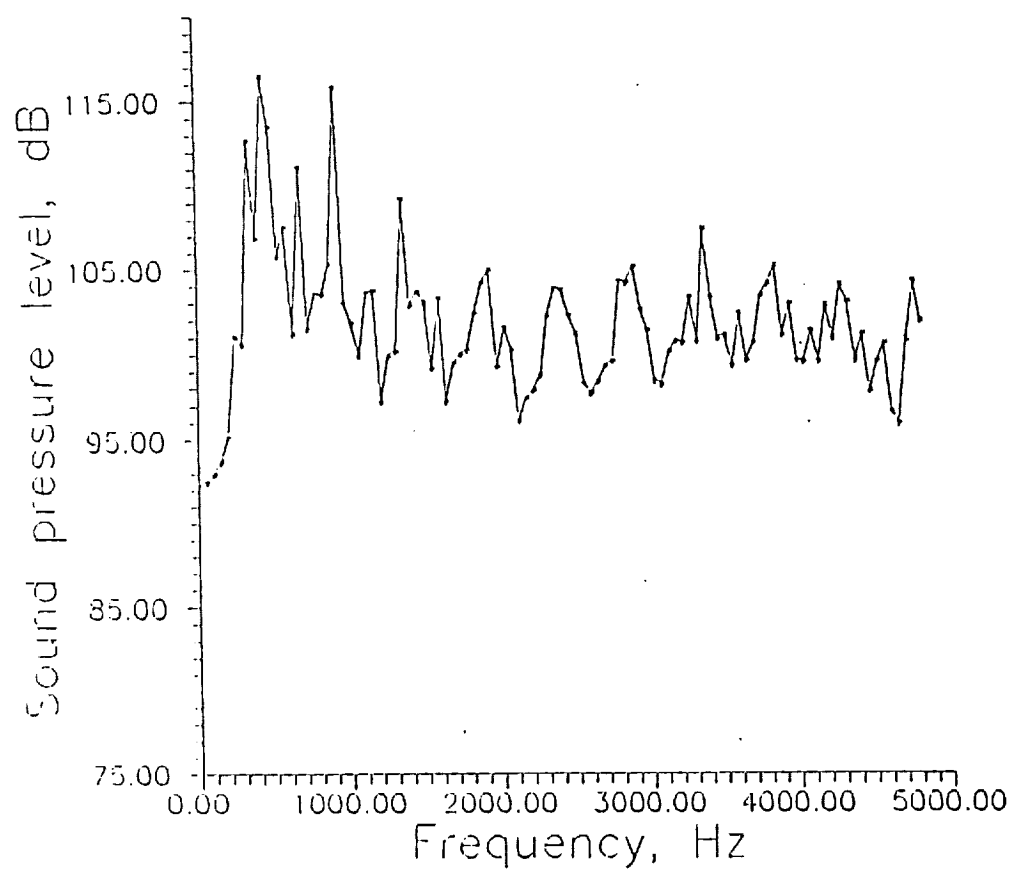


Figure 23. Tail Rotor Sound Pressure Level
Microphone 3

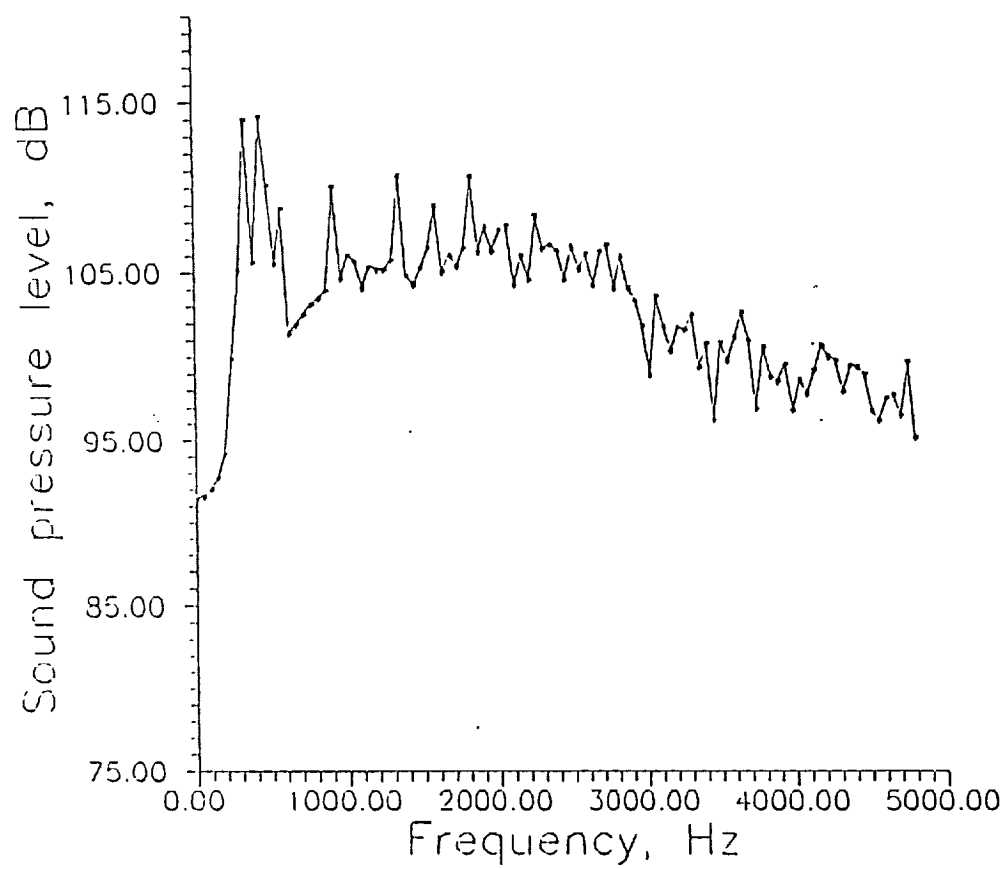


Figure 24. Tail Rotor Sound Pressure Level
Microphone 4

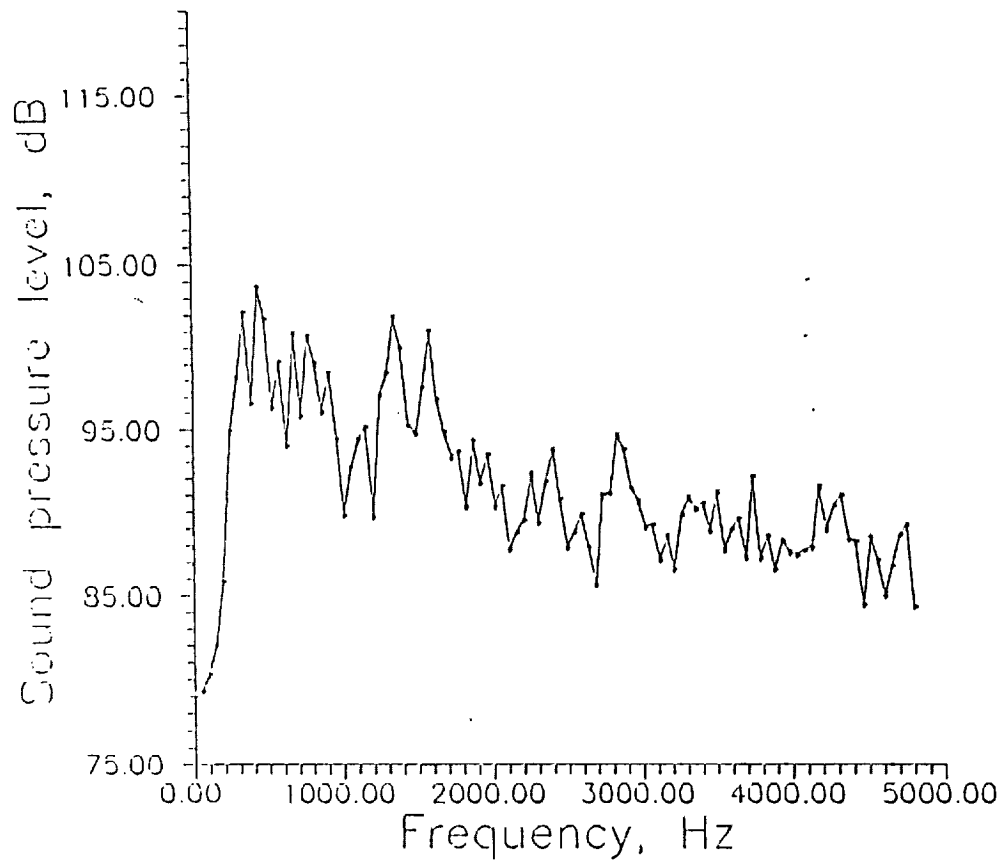


Figure 25. Tail Rotor Sound Pressure Level
Microphone 5

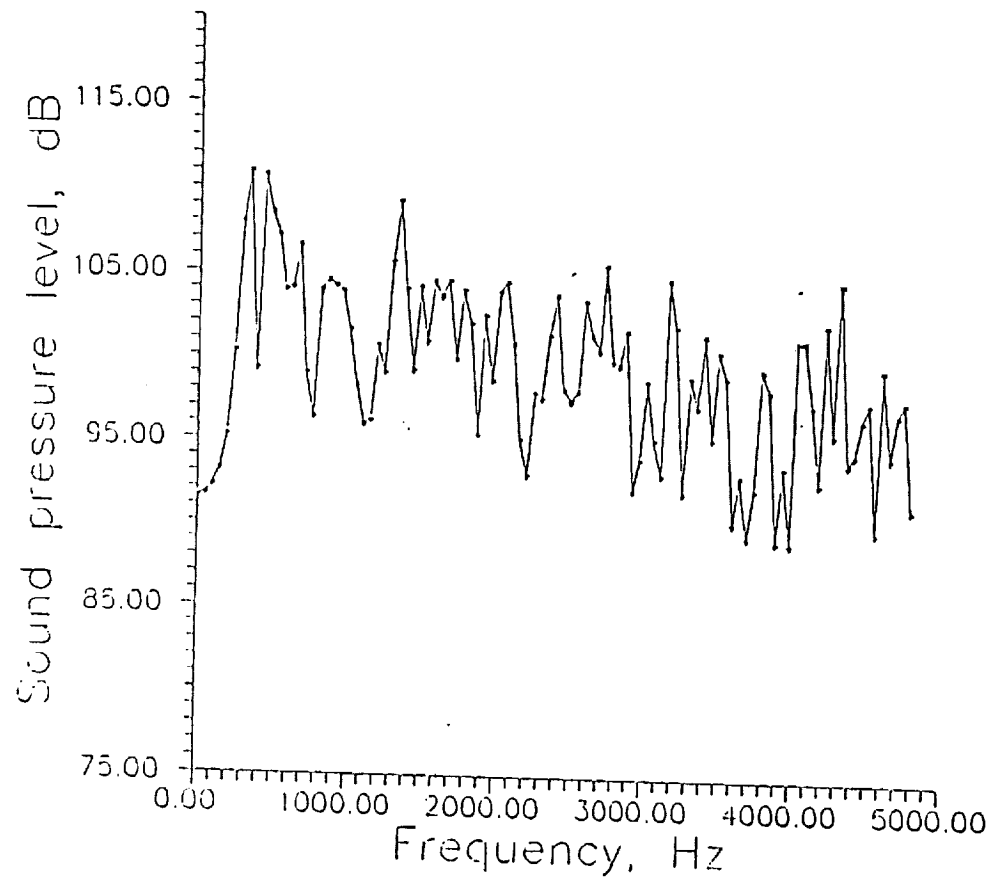


Figure 26. Tail Rotor Sound Pressure Level
with Random Shift Microphone 2

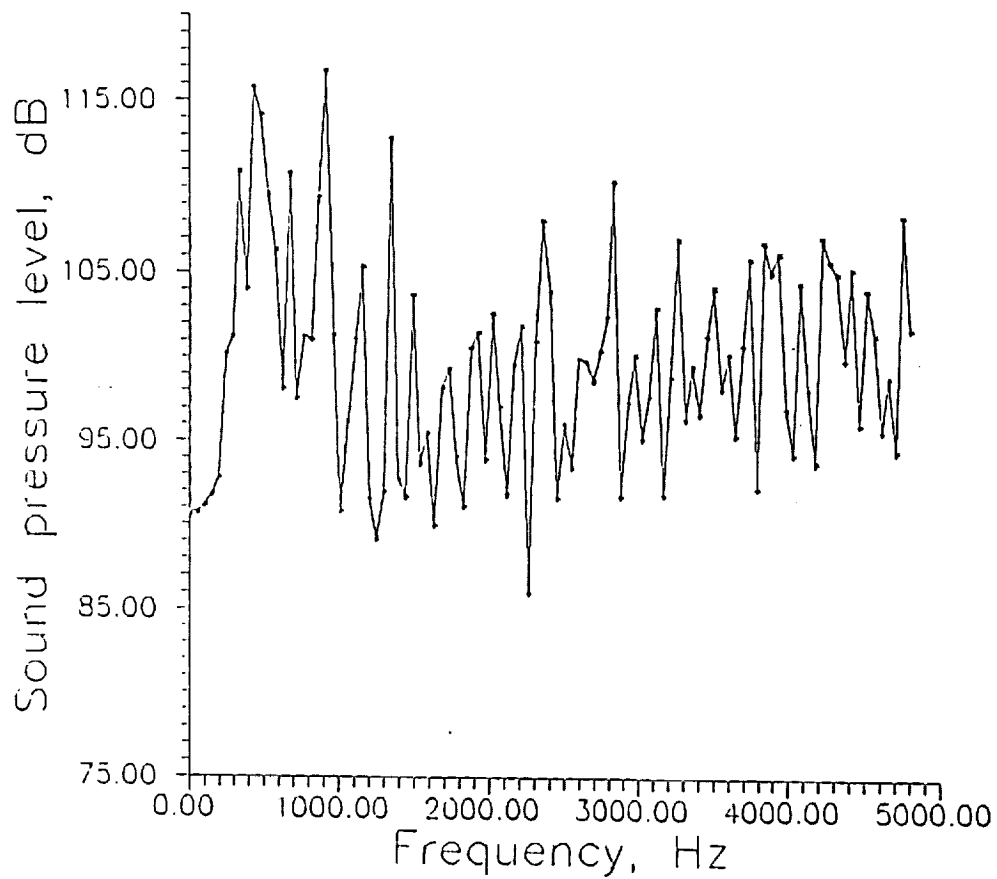


Figure 27. Tail Rotor Sound Pressure Level
with Random Shift Microphone3

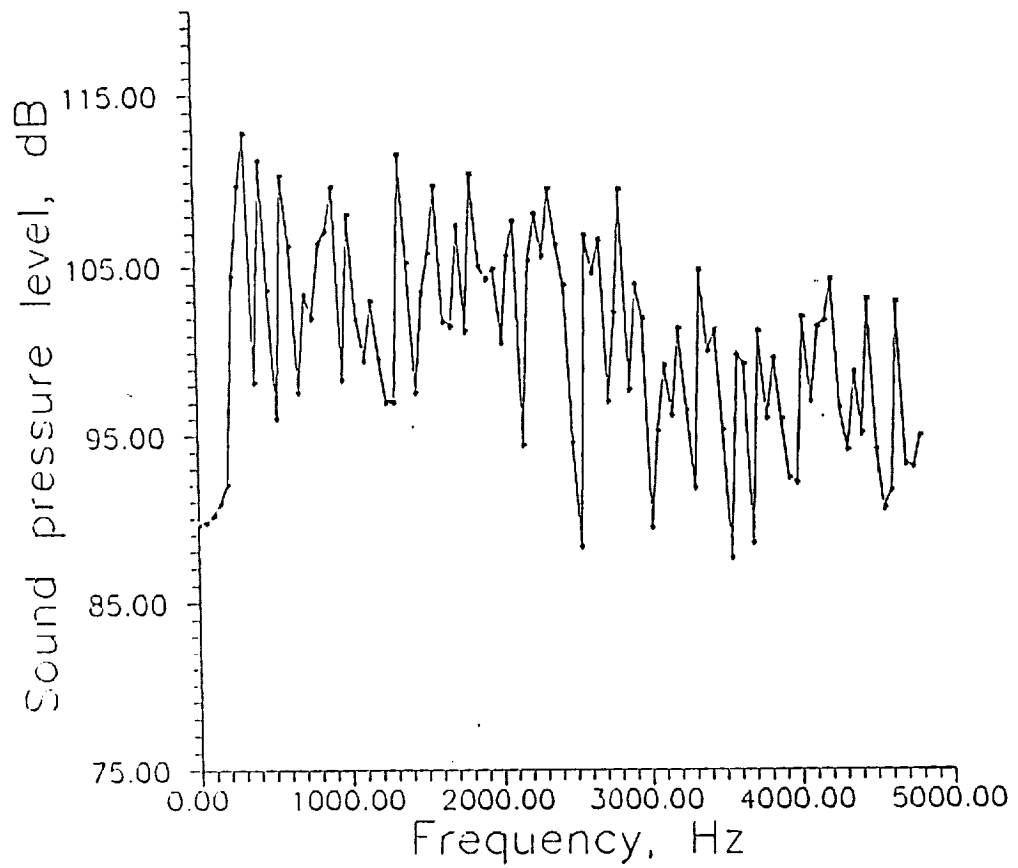


Figure 28. Tail Rotor Sound Pressure Level
with Random Shift Microphone 4

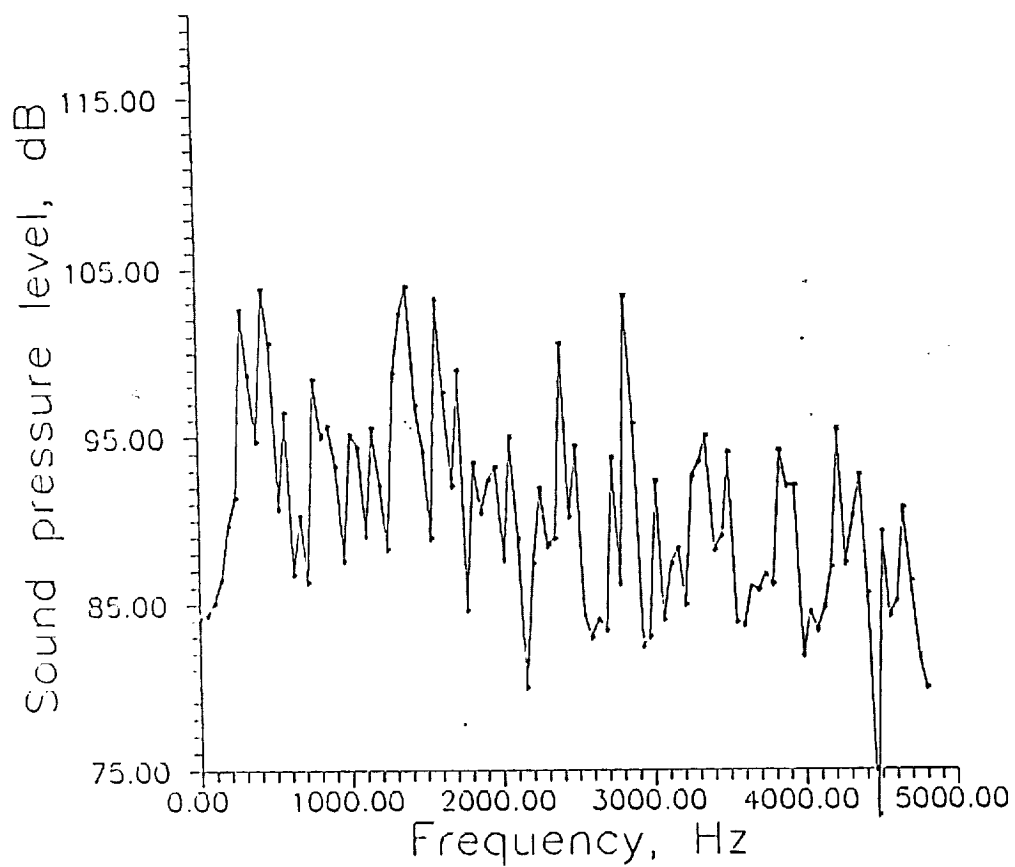


Figure 29. Tail Rotor Sound Preessure Level
with Random Shift Microphone 5

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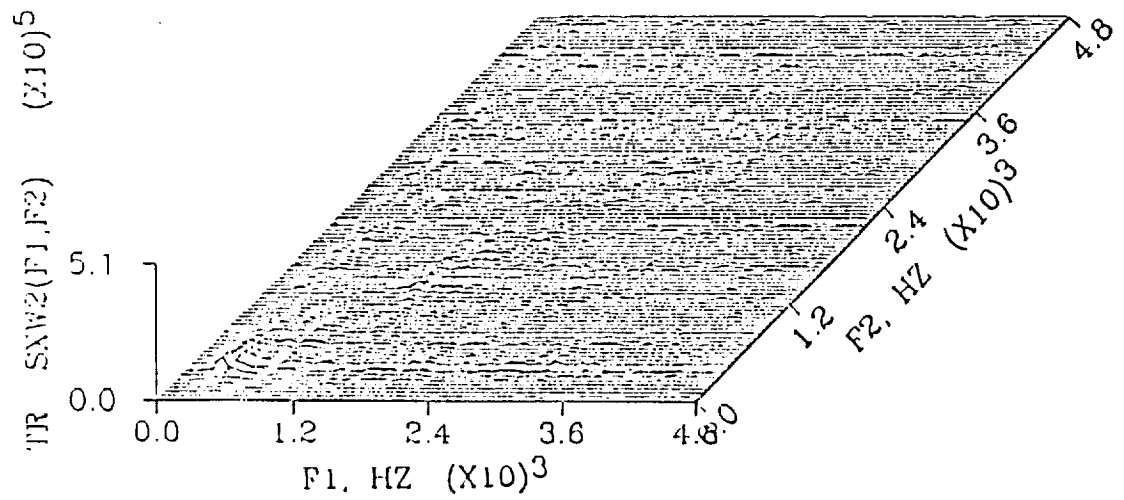


Figure 30. Tail Rotor Spectrum with Random Shift
Microphone 2

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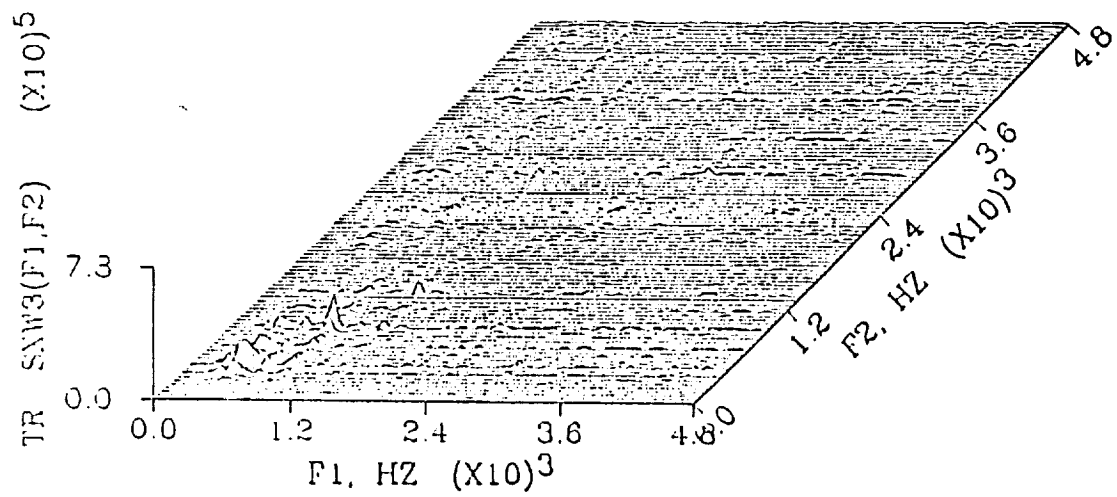


Figure 31. Tail Rotor Spectrum with Random Shift
Microphone 3

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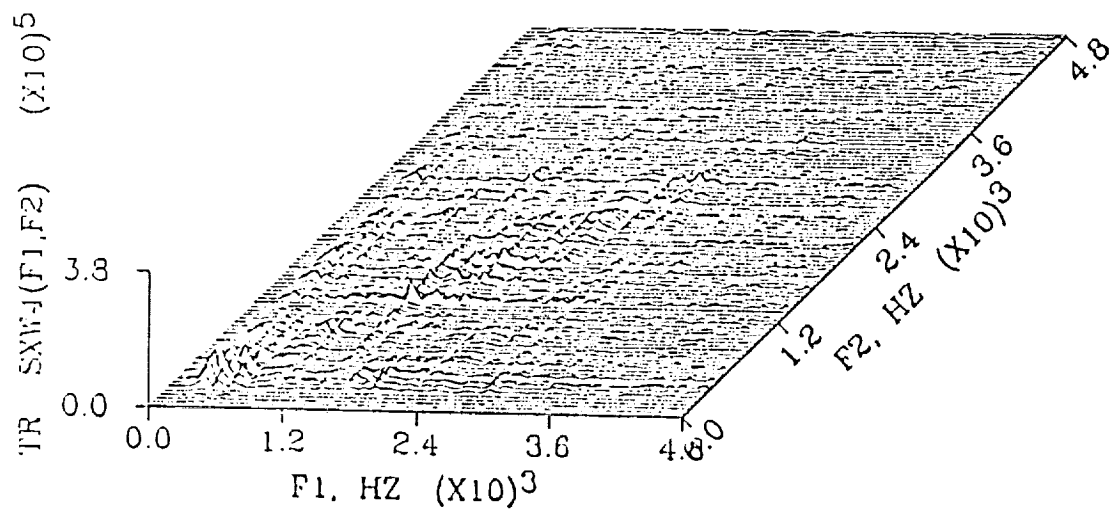


Figure 32. Tail Rotor Spectrum with Random Shift
Microphone 4

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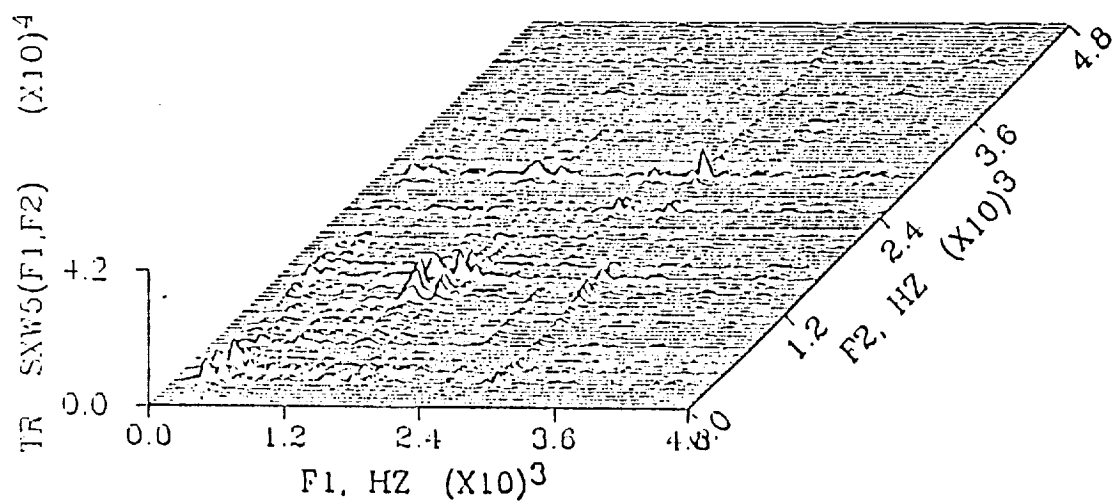


Figure 33. Tail Rotor Spectrum with Random Shift
Microphone 5

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APPENDICES

- A. 2-D Spectrum
- B. Spectral Coherence
- C. 1-D Spectrum

C234567

```
      INTEGER N,IWK(6150),S,Q
      REAL A(840),WK(6150)
      DIMENSION P(10000)
      COMPLEX XWT(0:500,11),SXW,SUM,SUM1
      COMPLEX X(1010)
      B=840
      S=840
      DO 6 N=1,10000
      READ(1,*)P(n)
6      CONTINUE
      Q=-839
      DO 40 J=1,11
      Q=Q+840
      K=0
      DO 8 I=Q,Q+839
      K=K+1
      A(K)=P(I)
8      CONTINUE
      CALL FFTRC (A,S,X,IWK,WK)
      ND2=S/2
      DO 20 I=2,ND2
      X(S+2-I)=CONJG(X(I))
20     CONTINUE
      DO 50 I=1,B/2+1
      XWT(I-1,J)=X(I)
50     CONTINUE
40     CONTINUE
      DO 90 Q=0,100
      DO 101 M=0,100
      SUM=(0.0,0.0)
      DO 70 J=1,11
      SUM=.0011108*(CONJG(XWT(M,J))*XWT(Q,J))+SUM
70     CONTINUE
      SUM1=SUM/11.0
      SXW=CABS(SUM1)
      WRITE(6,*)M,Q,SXW
101    CONTINUE
90     CONTINUE
      END
```

Appendix A. 2-D Spectrum

```

      INTEGER N,IWK(10152),S
      REAL SUM1,SUM2,SUM3
      REAL A(3334),WK(10152)
      DIMENSION U(10000)
      COMPLEX X(0:1668),SUM
      S=3334
      DO 6 N=1,3334
      READ(1,*)U(N)
6      CONTINUE
      DO 8 I=1,3334
      A(I)=U(I)
8      CONTINUE
      CALL FFTRC (A,S,X,IWK,WK)
      ND2=S/2
      DO 20 I=1,ND2
      X(S+2-I)=CONJG(X(I))
20     CONTINUE
      DO 7 I=1,S/2+1
      X(I-1)=X(I)
7      CONTINUE
      DO 200 Q=0,100,1
      DO 100 P=0,100,1
      SUM=(0.0,0.0)
      DO 10 M=0,7
      SUM=SUM + X(P+M)*CONJG(X(Q+M))
10     CONTINUE
      SUM1=((REAL(SUM))**2)+((AIMAG(SUM))**2)
      SUM2=0.0
      DO 21 M=0,7
      SUM2=SUM2 +((REAL(X(P+M)))**2) +((AIMAG(X(P+M)))**2)
21     CONTINUE
      SUM3=0.0
      DO 30 M=0,7
      SUM3=SUM3 +((REAL(X(Q+M)))**2) +((AIMAG(X(Q+M)))**2)
30     CONTINUE
      F=SUM1/(SUM2*SUM3)
      G=.35
      R=P*(20000.0/1667.0)
      W=Q*(20000.0/1667.0)
      if (F.ge.G)then
      WRITE(6,*)R,W
      ENDIF
100    CONTINUE
200    CONTINUE
      END

```

Appendix B. Spectral Coherence

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```
      INTEGER N,IWK(6150),S,Q
      REAL A(820),WK(6150)
      DIMENSION P(10000)
      COMPLEX XWT(0:500,11),SX,SUM,SUM1
      COMPLEX X(1010)
      B=820
      S=820
      DO 6 N=1,10000
      READ(1,*)P(n)
6      CONTINUE
      Q=-907
      DO 40 J=1,11
      Q=Q+908
      K=RAN(Q)*89
      N=0
      DO 8 I=Q,Q+907
      N=N+1
      IF(N.LE.820)THEN
      K=K+1
      A(N)=P(K)
      ENDIF
8      CONTINUE
      CALL FFTRC (A,S,X,IWK,WK)
      ND2=S/2
      DO 20 I=2,ND2
      X(S+2-1)=CONJG(X(I))
20     CONTINUE
      DO 50 I=1,B/2+1
      XWT(I-1,J)=X(I)
50     CONTINUE
40     CONTINUE
      DO 101 M=0,100
      SUM=(0.0,0.0)
      DO 70 J=1,11
      SUM=.0011108*(CONJG(XWT(M,J))*XWT(M,J))+SUM
70     CONTINUE
      SX=SUM/11.0
      SXW=REAL(SX)
      G=SXW
      SPL=10*LOG10(G)+74
      R=M*(20000/410)
      WRITE(6,*)R,SPL
101    CONTINUE
      END
```

Appendix C. 1-D Spectrum

ABSTRACT

A TWO DIMENSIONAL POWER SPECTRAL ESTIMATE FOR SOME NONSTATIONARY PROCESSES

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A two dimensional spectral estimate for a nonstationary process is developed. It addresses the need for modelling phenomena which exhibit nonstationary behavior. Currently these phenomena are usually modelled as stationary processes. The spectrum of a nonstationary process is two dimensional while the spectrum of stationary processes is one dimensional. Therefore, these phenomena are not as completely represented when modelled as stationary processes. The usual method of analyzing nonstationary processes as if they were stationary is compared to the two dimensional estimate which has been developed. In addition, a random phase shift which as the theory shows should produce a stationary process, is introduced to our nonstationary process. This stationary process can then be analyzed in the usual way. The results are compared with the methods described above.